PHASE SEQUENCE OPTIMIZATION IN A NETWORK OF MULTIPHASE SIGNALS

By

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This final product is dedicated to my parents,
Paul E. and Gustavia Johnson
and
my husband,
Ronnie A. Sabb.

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PHASE SEQUENCE OPTIMIZATION IN A NETWORK OF MULTIPHASE SIGNALS

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The purpose of this study is to develop a methodology which will determine the optimal phase sequences at each intersection of a network of multiphase signals. The cycle length and green time allocation are assumed to be already determined and are not optimized. This is accomplished in three stages. In the first stage an algorithm which determines the maximum sum of inbound and outbound green bands on a two signal arterial highwway is developed. This algorithm gives maximum bandwidths for each of sixteen possible phase sequence combinations at the signals of the arterial highway. In the second stage the algorithm is extended to optimize any arterial highway. Optimal phase sequence combinations and corresponding offsets are outputs of the algorithm. Finally, the selection of the best phase sequence combination for a closed network is found by using offsets, splits, and cycle length to formulate a function of several variables whose maximum value is the maximum sum of green bands on the network. A case study is used to demonstrate the selection technique.

The results of this study may be used to supplement existing traffic signal timing plan design techniques which do not include phase sequence selection capabilities.

CHAPTER ONE

INTRODUCTION

Traffic control has been an important area of focus for safety on roadways since the start of the automobile era. The earliest control devices were hand signals and these signals were later followed by traffic lights. While the date of the installation of the first electric signal in the United States is disputed (Cleveland in 1914[1], Detroit, Michigan[2], Texas in 1921[3]), the fact that signals are one of the most fundamental and important means of traffic control is not.

The evolution of traffic signals developed in two major ways: which were toward coordination and in the direction of traffic responsiveness. These evolutionary developments were designed to accomplish an objective other than the primary one of safety. Decreasing the inconvenience of unnecessary stops and delay caused by traffic signals and facilitating the full utilization of available capacity are two of their objectives.

Several control schemes and strategies have been developed to accomplish these objectives. These schemes include traffic signals, traffic management measures, one way streets, prohibition and route control devices. The control strategies include the principle of synchronization of a group of traffic signals. Synchronization is achieved by communication between individual intersection controllers.

The maximum through-band synchronization scheme is the oldest and most widely used synchronization scheme. It consists of offsetting the beginning of the green phases of successive controllers with respect to a system reference time to allow as many vehicles as possible to drive through these intersections at some design speed without stopping. Bandwidth, through-band and green-band are terms used to describe the number of seconds in each cycle in which a vehicle traveling the arterial street is able to theoretically start at the first signal and travel through to the last signal at the design speed without encountering a red indication.

The time space diagram is a tool used for the analysis of the flow of vehicles and measurement of bandwidth on an arterial street. The signals of the arterial highway are represented by vertical lines, rectangles superimposed over the vertical lines are used to represent red (darkened rectangle) and green (clear rectangle) periods, and lines of constant slope are used to represent the speed of the vehicles alone the arterial. One axis specifies time and the other axis specifies location (space). Figure 1.1

illustrates a typical time space diagram. Each direction of an arterial highway has its own bandwidth.

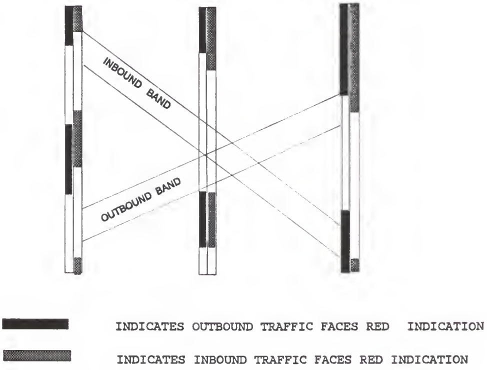

INBOUND BAND
OUTBOUND BAND
INDICATES OUTBOUND TRAFFIC FACES RED INDICATION
INDICATES INBOUND TRAFFIC FACES RED INDICATION

FIGURE 1.1: TYPICAL TIME-SPACE DIAGRAM

Background Discussion

There are a number of bandwidth maximization methods. These include graphical, algebraic and trial and error methods. These methods were limited to special cases such as constant block lengths, equal green bands at each intersection and constant progression speeds if the system was two-way. Analytical methods have been developed and programmed which have more flexibility. Synchronization is optimized generally by maximizing a weighted sum of through-bands and/or by minimizing a weighted sum of stops and delay. The timing components cycle length, splits, offsets and phase sequence are adjusted to attain the optimum objective function value in both synchronization methods.

Network control is a form of signal control for a group of adjacent signalized intersections where coordination is considered. The basic terms used in discussing signalized intersection control are 1) phase--the portion of a signal cycle allocated to any single combination of one or more traffic movements simultaneously receiving the right of way during one or more intervals, 2) sequence--a series of successive phase assignments during the cycle, 3) interval--a discrete portion of the signal cycle during which the signal indications remain unchanged, 4) cycle length--the time required for one complete sequence of signal intervals, 5) split--the percentage of a cycle length allocated to each of the various phases in a signal cycle, and 6) offset--the time difference between the start of the green indication at one intersection as related to the system time reference point. Figure 1.2 illustrates these components of signal phasing.

Although several signal phasing components were defined previously, this research concentrates on optimizing phase sequences. Phasing is a technique to reduce conflicts between traffic movements at signalized intersections.

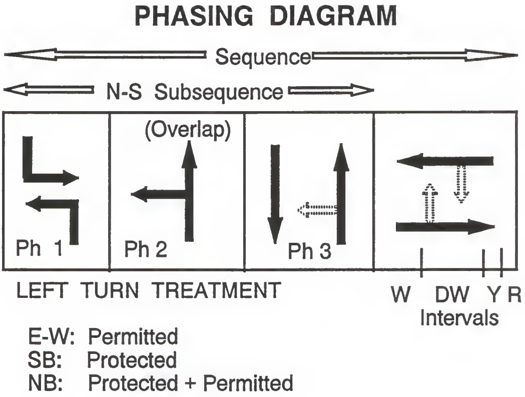


FIGURE 1.2: SIGNAL PHASING COMPONENTS

The phasing issue primarily relates to a left turn phasing problem. Left turn treatment alternatives include permitted, protected and permitted/protected. Permitted left turn treatment indicates that the left turning traffic is permitted to make the left turn whenever the driver judges that the gap in opposing traffic is sufficient. Protected treatment indicates that left turn traffic must wait for a green arrow to turn. Permitted/protected treatment on an approach indicates that both types of left turn treatments are used. The problem of determining the type of left turn treatment needed has been explored and warrants have been developed to aid the traffic engineer in making this decision[4].

In analyzing the problem of bandwidth maximization only the two opposing approaches are considered at a given time for each intersection. This is possible because the movements from the side approaches can not move simultaneously with the movements on the approaches under consideration. For notational purposes one of the directions on the arterial highway under consideration will be called the inbound direction and the opposing direction will be called the outbound direction. The following phase sequences use phase names to represent which movements have the right of way during the phase. The "O" designation means that both the outbound movements, through and left turning, have the green indication. "I" refers to the phase in which both inbound movements have the green, "T" represents the phase in which both through movements, inbound through and outbound through, have the right of way and "L" refers to the phase in which both left movements have the green indication. Figure 1.3 illustrates the movements allowed with each phase name described above.

The following phase sequences are possible: The first half of the name of each sequence reflects whether the outbound left turn precedes or follows the through movement and the second refers to the inbound left turn. The four phase sequences to be analyzed are as follows:

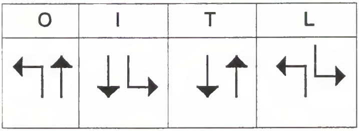


FIGURE 1.3: MOVEMENTS ALLOWED WITH PHASES

The Lead-lag Sequence

In this sequence the outbound left turn leads and the inbound left turn lags. The phase sequence is OTI which means outbound left and through have the green first, followed by a second phase where through movements have the green indication and in the final phase the inbound left and through movements get the right of way. Figure 1.4 illustrates this phase sequence.

The Lead-lead Sequence

Both the inbound and outbound left turns lead with this phase sequence. This phase sequence allows one of the directions to be assigned more time than the other. Consequently the phase sequence is either LIT or LOT. This phase sequence is diagrammed in figure 1.5.

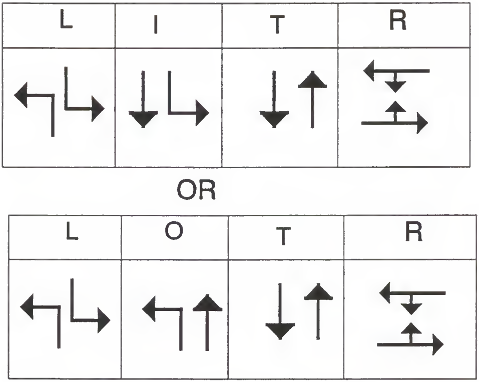


FIGURE 1.4: THE LEAD-LAG SEQUENCE

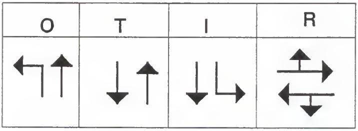


FIGURE 1.5: THE LEAD-LEAD SEQUENCE

The Lag-lead Sequence

The outbound left turn lags and the inbound left turn leads with the lag-lead sequence. In this case the phase sequence is ITO. Figure 1.6 illustrates this phase sequence.

The Lag-lag Sequence

Both left turns lag with lag-lag phasing. The phase sequence is TOL or TIL depending on which direction requires more time. This phase sequence is illustrated in figure 1.7.

Statement of the Problem

Various algorithms have been developed to optimize the timing of signalized traffic systems. Cycle length, splits and offset optimization algorithms for networks have been developed and have been programmed. Algorithms which completely optimize the timing components of arterial highways have also been developed. The existing algorithms for the determination of phase sequences in a network of multiphase signals, however, are limited.

Morgan and Little[5] developed a method of optimizing offsets on an arterial highway using the cycle length, splits and travel times on the arterial highway. Bleyl[6] developed a model for selecting cycle length(within ranges) and offsets which maximize the bandwidth on an arterial highway using travel times, volume and minimum green times.

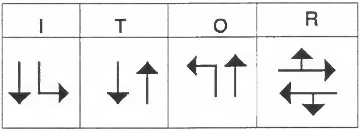


FIGURE 1.6: THE LAG-LEAD SEQUENCE

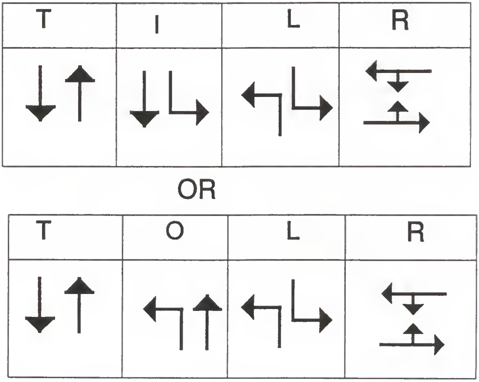


FIGURE 1.7: THE LAG-LAG SEQUENCE

Yardeni[7,8] developed a model which determines cycle length, splits and offsets on arterial highways. Brooks[9] developed a method which selects offsets, splits and cycle length at signals using the intersection with the shortest main street green (called the critical intersection) and a range of cycle lengths. Messer et al.[10] developed a computer program for Brook's algorithm. Hillier[11] developed an algorithm and Wagner and Gerlough[12] developed a computer program to select offsets on an arterial highway. Little et al[13] developed a model which optimizes phase sequences, cycle length and splits on arterial highways and triangular networks.

The algorithms for analyzing networks are much more limited and only one optimizes phase sequence combinations at the signals of the network. Robertson[14] developed an algorithm which determines optimal splits, and offsets in a multiphase network of signals. The algorithm has been programmed and is widely used in optimizing networks. While an optimal cycle length can be determine by multiple runs of the program, phase sequences must be input to the program and as such are not selected by the algorithm. Messer et al.[15] developed an algorithm which optimizes left turn phase sequences, cycle length, and offsets in signalized networks. The algorithm requires extensive data preparation and substantial computer resources.[16] Cohen[17] and Rogness[18] have shown that optimization of left turn phasing can significantly improve performance on signalized networks.

Thus this paper concentrates on the need to develop a method to determine the best phase sequence at each intersection of a multiphase network.

In this research the goal is to develop an algorithm which determines optimal phase sequences for the signals of a closed network which can be implemented using limited computer resources and which when combined with existing algorithms provides a complete optimization of a network of multiphase signals.

The problem, therefore, is to assign appropriate phase sequences to the signals of a multiphase network so that the overall inconvenience of unnecessary stops and delay is minimized and progression on the arteries of the network is maximized. The objective of this research is to develop an algorithm which selects optimal phase sequences at each intersection of a network of multiphase signals using the maximum sum of green band as the performance criterion and which requires minimal data input and computer resources.

Scope of the Study

This dissertation focuses on developing an algorithm which determines the optimal phase sequence combinations in a network of multiphase signals. The study does not attempt to present guidelines for selecting the appropriate type of left turn signal treatment or to optimize cycle length, green allocation or offsets.

Figure 1.8 depicts the analytical steps carried out in the design of a coordinated network of multiphase signals. Even though one can argue that the determination of green apportionment(splits) could be coincidental with phase sequence selection, it is assumed in this research that splits are known.

Gartner[19,20] has found that since both throughput capacity and delay time increase with cycle length, the common cycle time should be the minimum required to satisfy the intersection with the largest traffic volume. Other methods of determining network cycle time have been described by Webster[21] and in the Highway Capacity Manual[4]. For purposes of this research network cycle time is assumed to be determined.

The linking of signals is obtained by appropriate selection of offsets through the network. The network closure constraint requires that the sum of the offsets around each loop of the network be an integer multiple of the cycle length. Network optimization programs TRANSYT-7F[22], MAXBAND-86[15] and SIGOP II[23,24] all provide good solutions to the offset optimization problem. Offset optimization is not within the scope of this research effort.

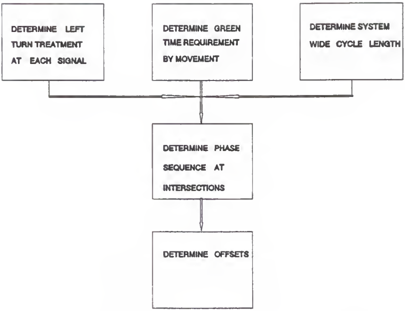

DETERMINE LEFT TURN TREATMENT AT EACH SIGNAL
DETERMINE GREEN TIME REQUIREMENT BY MOVEMENT
DETERMINE SYSTEM WIDE CYCLE LENGTH
DETERMINE PHASE SEQUENCE AT INTERSECTIONS
DETERMINE OFFSETS

FIGURE 1.8: STEPS IN THE DESIGN OF A COORDINATED NETWORK OF MULTIPHASE SIGNALS

To accomplish the goals of this research, the following analytical steps were taken.

1) Develop a model which, given the green requirement per movement, determines the green time per phase for each phase sequence.
2) Develop a method of determining the beginning and end of green at an intersection as a function of the offset at the intersection for each phase sequence.
3) Develop a closed form expression for the outbound and inbound bandwidth between pairs of consecutive

intersections as a function of the offset at the intersections.

4) Develop a method of determining the offset values which yield maximum value of the sum of the bandwidth functions developed under objective 3.
5) Develop an algorithm to determine the phase sequence pattern which yields the maximum attainable green-band on an artery using the values obtained in objective 4.
6) Develop a method of putting solutions to the arteries of a network together so as to optimize the sequences for the entire network.

These objectives require explicit consideration of several complex coordination problems.

Organization

In chapter two a step by step procedure for determining the maximum sum of inbound and outbound bands on a two intersection arterial highway is developed. In chapter three a discussion of a method of successively treating the constraints imposed by each signal of the arterial highway which allows the extension of the two signal arterial highway solution to any arterial highway is given. Chapter four provides a means of extending the arterial solution to a general closed network. Conclusions and recommendations are given in chapter five.

CHAPTER TWO

DEVELOPMENT OF AN ALGORITHM FOR THE TWO SIGNAL ARTERIAL HIGHWAY

In this chapter an algorithm to determine the optimal solution to systems consisting of two signals is developed. The first step is to determine the number of seconds of green to assign to each phase. The green per phase assignments are then used to determine the beginning and end of the green time at each intersection. Using these values the bandwidth between two signals is expressed as a function of the offset at the second signal of the system. Finally, a closed form solution to the problem of finding the maximum sum of the outbound and inbound bandwidths is developed.

Determination of Green Time Per Phase at a Single Intersection

In general, coordinated systems operate best when the main street flow is predominantly through traffic. Therefore to achieve the goal of maximizing the sum of the inbound and outbound bandwidths the objective in assigning green time to the phases is to assign as much time to the through movements as possible. The number of seconds of green to be allocated to each phase can be determined using the green time requirement per movement. The movements are "OBL", representing the outbound left turn green time requirement, "OBT" representing the outbound through movement green time, "IBL" representing inbound left turn green and "IBT" representing the inbound through green time requirement. These representations allow the following assignments to be made for each of the four phase sequences previously considered.

1) For lead-lag phasing, the phase sequence is identified as OTI which requires an assigned time to the outbound movement O, the through movement T, and the inbound movement I. With this phase sequence the outbound left traffic is served only during the O phase. Therefore the number of seconds assigned to the O phase must be at least OBL. In order to achieve the goal of assigning as much time to the through movement as possible, the decision made here is to limit the duration of the O phase to be exactly OBL. Similarly since the inbound left turn traffic is only served during the I phase, the decision is to assign a number of seconds at least equal to IBL to the I phase and to limit the duration so that it is exactly equal to IBL. Hence, the assignment of green time to the phases for lead- lag phasing are as follows:

O = OBL = number of seconds of green required for the outbound left turning traffic

I = IBL = number of seconds of green required for the inbound left turning traffic

$T = C_m - OBL - IBL$

If $T < 0$ then let $O = 0$, $I = 0$, and $T = C_m$ where C_m is the number of total seconds allocated to the outbound and inbound movements on the artery.(Note $C - C_m$ is the total number of seconds allocated to the cross street movements at the intersection.) The "If" statement causes the left turn protection to be eliminated in the event that the total number of seconds allocated to the artery is insufficient.

2) For lag-lead phasing the assignments of green time per phase are made the same as for lead-lag phasing. 3) For lead-lead and 4) lag -lag phasing the phases used are L,O,I, and T. The assignment of green time to each phase is made with the goal of maximizing the number of seconds assigned to the through movement. The number of seconds assigned to each phase is

$L = \min [OBL, IBL]$

$I = \max [0, IBL - L]$

$O = \max [0, OBL - L]$

$T = C_m - L - I - O$

If $T < 0$ then set $I = 0$, $O = 0$, $L = O$ and $T = C_m$.

Determining the Beginning and End of Green

The key requirements in calculating the bandwidth on an arterial highway are the determination of the beginning and end of the green period at each intersection. The beginning and end of green at a signal depend on the offset, phase sequence and green time per phase.

Let a_o represent the beginning of the outbound green,

b_o the end of the outbound green,

a_i the beginning of the inbound green, and

b_i the end of the inbound green.

These representations determine the beginning and end of green in each phase sequence in the following ways:

1) Lead-Lag Phasing

Figure 2.1 illustrates the phases of the lead-lag phase sequence. The inbound and outbound directions are labeled on the right of the figure. The outbound green begins at the offset and continues until the time that the inbound left turn traffic is given the green indication. The I phase begins at this time and is labeled b_o on the figure. The inbound green begins at the time labeled a_i. The inbound time represents the time that the outbound left turning traffic is stopped and the inbound through traffic is then given the green indication. The inbound green continues through the I phase and ends at the point labeled b_i where the cross street traffic is given the green and all traffic on the arterial highway is stopped.

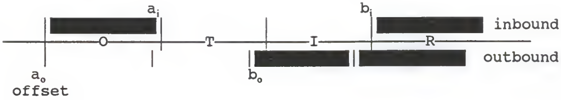
For the lead-lag sequence a_o, b_o, a_i, and b_i are computed as follows:

a_o = offset

b_o = offset + O + T

a_i = offset + O

b_i = offset + O + I + T



note: ■ indicates that the through traffic faces a red indication

FIGURE 2.1: BEGINNING AND END OF GREEN FOR LEAD-LAG PHASE SEQUENCE

The values of a_o, b_o, a_i, and b_i are determined similarly for the remaining three phase sequences using figures 2.2, 2.3 and 2.4. The resulting values are as follows:

2) Lead-Lead Phasing

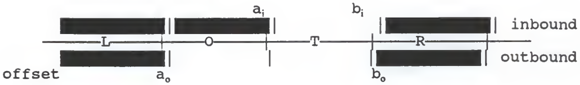
With this phase sequence either O=0 or I=0. In either situation the expressions below will give the correct values for each of the phases a_o, b_o, b_i, and b_i as illustrated in figure 2.2.

a_o= offset + L + I

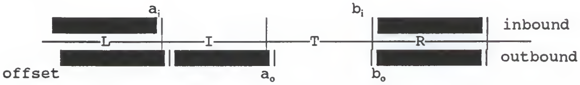
b_o = offset + L + I + O + T

a_i = offset + L + O

b_i = offset + L + O + I + T



OR



note: ■ indicates that the through traffic faces a red indication

FIGURE 2.2: BEGINNING AND END OF GREEN FOR LEAD-LEAD PHASE SEQUENCE

3) Lag-Lead Phasing

a_o= offset + I

b_o = offset + I + O + T

a_i = offset

b_i = offset + I + T

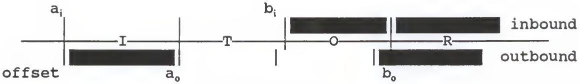
4) Lag-Lag Phasing

a_o= offset

b_o = offset + T+ O

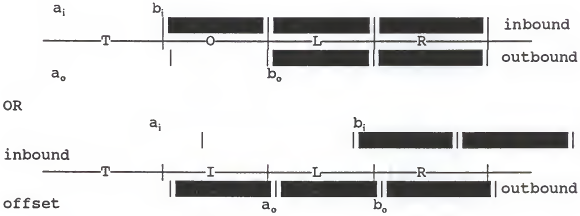
a_i = offset

b_i = offset + T + I



note: █ indicates that the through traffic faces a red indication

FIGURE 2.3: BEGINNING AND END OF GREEN FOR LAG-LEAD PHASE SEQUENCE



note: █ indicates that the through traffic faces a red indication

FIGURE 2.4: BEGINNING AND END OF GREEN FOR LAG-LAG PHASE SEQUENCE

Determining the Bandwidth Between Two Signals

The bandwidth between two signals can be computed from the beginning and end of the green at each of the two signals.

Let $a_{(o,1)}$ represent the beginning of the outbound green at signal 1,

$b_{(o,1)}$ the end of the outbound green at signal 1

$a_{(i,1)}$ the beginning of the inbound green at signal 1

$b_{(i,1)}$ the end of the inbound green at signal 1 and

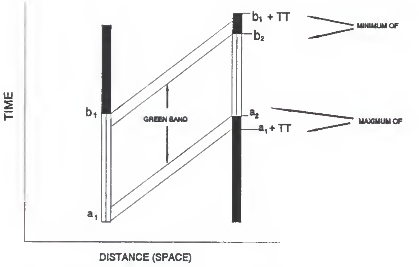
N the offset at the second signal.

($a_{(o,2)}$, $b_{(o,2)}$, $a_{(i,2)}$, and $b_{(i,2)}$ are similarly defined respectively for signal 2). Also let TTO represent the outbound travel time between signals 1 and 2 and let TTI represent the inbound travel time between the two signals. Figure 2.5 depicts how these values together with the travel time can be used to determine the bandwidth.

Figure 2.5 shows that the bandwidth is given by EOB - BOB, where EOB = end of band = min [$b_{(o,1)}$, $b_{(o,2)}$ -TT] and BOB = beginning of band = max [$a_{(o,1)}$, $a_{(o,2)}$ - TT].

The outbound and inbound bandwidth between any pair of signals can be computed from the beginning and end of green at each of the signals along with the travel time between the two signals.

The next step is to examine each of the possible sixteen combinations of phase sequences at the two signals. This examination gives the following outbound and inbound



GREEN BAND = MIN { b_1 + TT, b_2 } - MAX { a_1 + TT, a_2 }

FIGURE 2.5 TIME SPACE DIAGRAM SHOWING THE BANDWIDTH BETWEEN TWO SIGNALS

calculations. For convenience let the offset at the first signal be zero. Then the following values are defined for combinations of phase sequences at the two intersections.

Case 1) Lead-Lag at Signal 1 / Lead-Lag at Signal 2

AT SIGNAL 1	AT SIGNAL 2
$a_{(o,1)} = 0$	$a_{(o,2)} = N$
$b_{(o,1)} = O_1 + T_1$	$b_{(o,2)} = N + O_2 + T_2$
$a_{(i,1)} = O_1$	$a_{(i,2)} = N + O_2$
$b_{(i,1)} = T_1 + I_1 + O_1$	$b_{(i,2)} = N + T_2 + I_2 + O_2$

The outbound beginning of the green band denoted here as B_o is

$$B_o = \max\,[OB_1, OB_2 - TTO]$$

$$= \max\ [0,\ N-TTO].$$

The outbound end of the green band denoted here as E_o is

$$E_o = \min\ [b_{(o,1)},\ b_{(o,2)} - TTO]$$
$$= \min\ [O_1+T_1,\ N+O_2+T_2-TTO].$$

If $O_1 + T_1 > O_2 + T_2$, then the numbers TTO and $TTO+O_1+T_1-O_2-T_2$ divide the number line into three parts as shown in figure 2.6. The values of B_o and E_o are determined for values of the offset, N, within each section of the number line as indicated in figure 2.6.

Using the expressions for B_o and E_o one may then obtain expressions for the outbound bandwidth, OBW. The expressions are shown in figure 2.7 for each value of N, the offset at signal 2. Therefore the outbound bandwidth can be expressed as a function of N as

$$OBW(N) = \begin{cases} N + O_2 + T_2 - TTO & N < TTO \\ O_2 + T_2 & TTO \le N \le TTO+O_1+T_1-O_2-T_2 \\ O_1+T_1+TTO-N & N > TTO+O_1+T_1-O_2-T_2. \end{cases}$$

If $O_1+T_1 < O_2+T_2$, then the number line is again divided into three sections as shown in figure 2.8 and the values of B_o and E_o are as shown in the figure. Using these values the value of the outbound bandwidth in this case is illustrated in figure 2.9 and can be written in function form as

$$OBW(N) = \begin{cases} N + O_2 + T_2 - TTO & N < TTO+O_1+T_1-O_2-T_2 \\ O_1 + T_1 & TTO+O_1+T_1-O_2-T_2 \le N \le TTO \\ O_1 + T_1 + TTO - N & N > TTO \end{cases}$$

Now, let $A = \min [\ TTO\ ,\ TTO+O_1+T_1-O_2-T_2\ ]$

$B = \max [\ TTO\ ,\ TTO+O_1+T_1-O_2-T_2\ ]$

and $C = O_2 + T_2 - TTO$.

Then the outbound bandwidth function OBW is defined as follows:

$$OBW(N) = \begin{cases} N+C & N < A \\ A+C & A \leq N \leq B \\ A+B+C-N & N > B. \end{cases}$$

The inbound bandwidth function is determined using a similar analysis of the values of N and is as follows:

If $I_1+T_1 > I_2+T_2$, then

$$IBW(N) = \begin{cases} N+I_2+T_2-TTI+O_2-O_1 & N < TTI+O_1-O_2 \\ I_2+T_2 & TT+O_1-O_2 \leq N \leq TTI+O_1-O_2+I_1+T_1-I_2-T_2 \\ O_1+I_1+T_1+TTI-O_2-N & N > TTI+O_1-O_2+I_1+T_1-I_2-T_2 \end{cases}$$

If $I_1+T_1 < I_2+T_2$, then

$$IBW(N) = \begin{cases} N+I_2+T_2-TTI+O_2-O_1 & N < TTI+O_1-O_2+I_1+T_1-I_2-T_2 \\ I_1+T_1 & TTI+O_1-O_2+I_1+T_1-I_2-T_2 \leq N \leq TTI+O_1-O_2 \\ O_1+I_1+T_1+TTI-O_2-N & N > TTI+O_1-O_2 \end{cases}$$

Now if A, B, and C are defined as

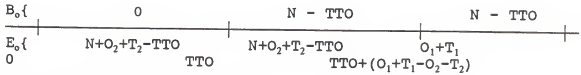
$A = \min [TTI+O_1-O_2,\ TTI+O_1-O_2+I_1+T_1-I_2-T_2]$

$B = \max [TTI+O_1-O_2,\ TTI\ +O_1-02+I_1+T_1-I_2-T_2]$

$C = O_2 + I_2 + T_2 - O_1 - TTI$

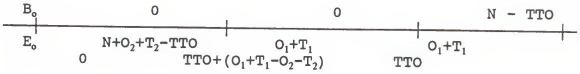
again the bandwidth function can be expressed as

$$IBW(N) = \begin{cases} N+C & N < A \\ A+C & A \leq N \leq B \\ A+B+C-N & N > B. \end{cases}$$



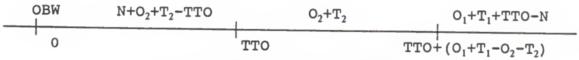
a) IF $TTO \leq TTO+(O_1+T_1-O_2-T_2)$

OR

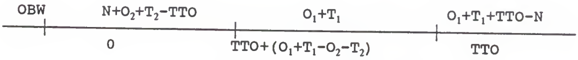


b) IF $TTO > TTO+(O_1+T_1-O_2-T_2)$

FIGURE 2.6: EXPRESSIONS FOR THE BEGINNING AND END OF THE OUTBOUND BAND LEAD-LAG PHASE SEQUENCE



a) IF $TTO \leq TTO+(O_1+T_1-O_2-T_2)$



b) IF $TTO > TTO+(O_1+T_1-O_2-T_2)$

FIGURE 2.7: VALUE OF THE OUTBOUND BANDWIDTH FOR LEAD-LAG PHASE SEQUENCE

Thus both the outbound and inbound bandwidth functions fit a general form as illustrated above. The next case to be examined is one where the phase sequence at signal 1 is lead-lag and the phase sequence at signal 2 is lead-lead. The bandwidth functions are calculated as shown below and fit into the same general form as the preceding case.

Case 2) Lead-Lag/Lead-Lead Phasing

AT SIGNAL 1	AT SIGNAL 2
$a_{(o,1)} = 0$	$a_{(o,2)} = N + L_2 + I_2$
$b_{(o,1)} = O_1 + T_1$	$b_{(o,2)} = N + L_2 + I_2 + O_2 + T_2$
$a_{(i,1)} = O_1$	$a_{(i,2)} = N + L_2 + O_2$
$b_{(i,1)} = T_1 + I_1$	$b_{(i,2)} = N + L_2 + O_2 + I_2 + T_2$

The outbound beginning of band, $B_o = \max[OB1, OB2 - TTO]$

$= \max[0, N+L_2+I_2-TTO]$.

The outbound end of band, $E_o = \min[b_{(o,1)}, b_{(o,2)}-TTO]$

$= \min[O_1+T_1, N+L_2+I_2+O_2+T_2-TTO]$.

Thus if $O_1 + T_1 > O_2 + T_2$, then the values of B_o and E_o are as indicated in figure 2.8 and the value of the outbound bandwidth is as in figure 2.9. The functional representation of the bandwidths are as follows:

$$OBW(N) = \begin{cases} N+L_2+I_2+O_2+T_2-TTO & N < TTO-L_2-I_2 \\ O_2 + T_2 & TTO-L_2-I_2 \leq N \leq TTO-L_2-I_2+O_1+T_1-O_2-T_2 \\ O_1+T_1+TTO-L_2-I_2-N & N > TTO-L_2-I_2+O_1+T_1-O_2-T_2 \end{cases}$$

and if $O_1+T_1 < O_2 + T_2$, then

$$\mathrm{OBW}(N) = \begin{cases} N+L_2+I_2+O_2+T_2-TTO & N < TTO-L_2-I_2+O_1+T_1-O_2-T_2 \\ O_1 + T_1 & TTO-L_2-I_2+O_1+T_1-O_2-T_2 \le N \le TTO-L_2-I_2 \\ O_1+T_1+TTO-L_2-I_2-N & N > TTO-L_2-I_2 \end{cases}$$

The inbound bandwidth IBW is computed as follows:

If $O_1+I_1+T_1 > I_2+T_2$, then

$$\mathrm{IBW}(N) = \begin{cases} N+L_2+O_2+I_2+T_2-TTI-O_1 & N < TTI+O_1-L_2-O_2 \\ I_2+T_2 & TTI+O_1-L_2-O_2 \le N \le TTI+O_1-L_2-O_2+I_1+T_1-I_2-T_2 \\ O_1+I_1+T_1-L_2-O_2+TTI-N & N > TTI+O_1-L_2-O_2+I_1+T_1-I_2-T_2. \end{cases}$$

If $I_1+T_1 < I_2+T_2$, then

$$\mathrm{IBW}(N) = \begin{cases} N+L_2+O_2+I_2+T_2-TTI-O_1 & N < TTI+O_1-L_2-O_2+I_1+T_1-I_2-T_2 \\ I_1+T_1 & TTI+O_1+I_1+T_1-I_2-T_2 \le N \le TTI+O_1 \\ O_1+I_1+T_1+TTI-N & N > TTI+O_1. \end{cases}$$

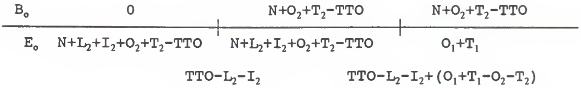


FIGURE 2.8: EXPRESSIONS FOR THE BEGINNING AND END OF OUTBOUND BANDWIDTH FOR LEAD-LEAD PHASE SEQUENCE

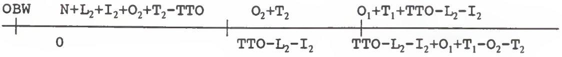


FIGURE 2.9: VALUE OF THE OUTBOUND BANDWIDTH FOR LEAD-LEAD PHASE SEQUENCE

General Form of the Bandwidth Function

The outbound and inbound bandwidth functions are all of the general form discussed below.

Let $F_i(x)$ be a piecewise linear function defined by

$$F_i(X) = \begin{cases} X + C_i & X < A_i \\ A_i + C_i & A_i \leq X \leq B_i \\ A_i + C_i + B_i - X & X > B_i. \end{cases}$$

The maximum value of this type function is easily seen to be attained at any x in the interval $[A_i, B_i]$ and is equal to $A_i + C_i$. Figure 2.12 illustrates what the graph of a general bandwidth function looks like.

The solution to the maximum sum of two such functions depends on the relationship of the A_i's to the B_i's. There are six cases to consider (since $A_i \leq B_i$).

CASE 1. $A_1 \leq B_1 \leq A_2 \leq B_2$

CASE 2. $A_1 \leq A_2 \leq B_1 \leq B_2$

CASE 3. $A_1 \leq A_2 \leq B_2 \leq B_1$

CASE 4. $A_2 \leq B_2 \leq A_1 \leq B_1$

CASE 5. $A_2 \leq A_1 \leq B_2 \leq B_1$

CASE 6. $A_2 \leq A_1 \leq B_1 \leq B_2$

The maximum value of the sum of F_1 and F_2 can then be determined for each case. Figure 2.13 illustrates the solution for case 1. The sum , F_1+F_2, is defined in this case as follows:

$$(F_1+F_2)(X) = \begin{cases} 2X + C_1 + C_2 & X \leq A_1 \\ X + A_1 + C_1 + C_2 & A_1 \leq X \leq B_1 \\ A_1 + B_1 + C_1 + C_2 & B_1 \leq X \leq A_2 \\ A_1 + B_1 + C_1 + A_2 + C_2 - X & A_2 \leq X \leq B_2 \\ A_1 + B_1 + C_1 + A_2 + B_2 + C_2 - 2X & X >= B_2. \end{cases}$$

The maximum value of the sum is given at X in the interval $[B_1, A_2]$ with the value of the sum equal to $A_1+B_1+C_1+C_2$. The maximum for each case described above is shown in the table below.

Case	Maximum Sum	optimal X
1. $A_1 \leq B_1 \leq A_2 \leq B_2$	$C_1 + C_2 + A_1 + B_1$	$B_1 \leq X \leq A_2$
2. $A_1 \leq A_2 \leq B_1 \leq B_2$	$C_1 + C_2 + A_1 + A_2$	$A_2 \leq X \leq B_1$
3. $A_1 \leq A_2 \leq B_2 \leq B_1$	$C_1 + C_2 + A_1 + A_2$	$A_2 \leq X \leq B_2$
4. $A_2 \leq B_2 \leq A_1 \leq B_1$	$C_1 + C_2 + A_2 + B_2$	$B_2 \leq X \leq A_1$
5. $A_2 \leq A_1 \leq B_2 \leq B_1$	$C_1 + C_2 + A_1 + A_2$	$A_1 \leq X \leq B_2$
6. $A_2 \leq A_1 \leq B_1 \leq B_2$	$C_1 + C_2 + A_1 + A_2$	$A_1 \leq X \leq B_1$

For each of the sixteen phase sequence combinations the maximum sum of inbound and outbound bandwidths is computed using the solution for the general function form and the definitions of the constants A_i, B_i, and C_i for each of the phase sequence combinations. The result is that the maximum sum of the inbound and outbound bands is determined for each of the sixteen phase sequence combinations.

Thus the maximum sum of inbound and outbound bandwidth between any two signals can be determined by first defining the outbound bandwidth function and the inbound bandwidth function. After defining the two functions the maximum sum can be found as demonstrated above.

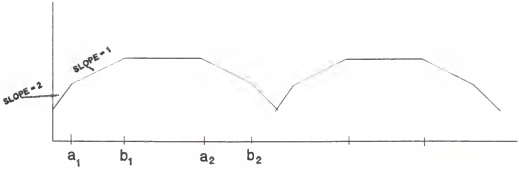


FIGURE 2.10: GENERAL BANDWIDTH FUNCTION

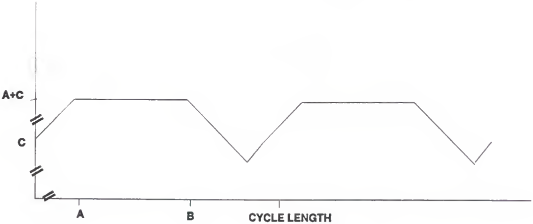


FIGURE 2.11: SUM OF TWO BANDWIDTH FUNCTIONS

With phase sequence combination 1) lag-lag/lag-lag, for example, $A_1 = TTO$,

$B_1 = TTO+T_1+O_1-T_2-O_2$,

$C_1 = T_2+O_2-TTO$

$A_2 = TTI$

$B_2 = T_1+I_1+TTI-T_2-I_2$

$C_2 = T_2+I_2-TTI$.

The expressions for the inbound and outbound bandwidth for each of the remaining possible phase sequence combinations are derived similarly. The expressions for these functions depend on the phase sequences at each end of the artery. Values for A_i, B_i, and C_i for each of the sixteen possible phase sequences are shown in appendix A.

SUMMARY OF AN ALGORITHM OPTIMIZING A TWO SIGNAL SYSTEM

The optimal phase sequence combinations for each signal of a two-signal artery can be summarized in the following steps.

Step 1. Do the following at each signal

Do the following for each of four phase sequences (lead-lag, lead-lead, lag-lead, and lag-lag).

a. Assign green time to each phase using formulas 2.1, 2.2, 2.3, and 2.4.

b. Determine the beginning and end of the green as functions of the offset at signal 2 using formulas 2.5, 2.6, 2.7 and 2.8.

Step 2. Do the following for each combination of phase sequences at signals 1 and 2.

i. Express the inbound and outbound bandwidths as functions of the offset at signal 2 using the beginning and end of green functions obtained in step 1b.

ii. Find the maximum sum of the inbound and outbound bandwidth functions obtained in step 2i.

Step 3. Select the phase sequence which has the maximum sum of bands as determined in step 2.

CHAPTER THREE

EXPANDING THE ALGORITHM TO THE GENERAL ARTERIAL HIGHWAY

Optimal phase sequence combinations for an arterial highway system are obtained as a natural extension of the two-signal solution discussed previously. In this chapter an algorithm is developed to obtained the maximum sum of bandwidths by successively including the signals of the artery.

For an arterial highway consisting of N signals, $S_1, S_2, \ldots,$ and S_N, let x_i denote the offset at signal S_i. The discussion in chapter 2 above shows that the beginning of the green at each signal can be expressed as $x_i + c_i$ for some constant c_i. The value of c_i depends on which of the four phase sequences is chosen at the signal. Likewise the end of the green at each signal can be expressed as $x_i + k_i$ for some constant k_i.

Now, let TT(m,n) represent the travel time from signal m to signal n, $e_i = x_i + k_i$ represent the end of the green at signal S_i, and $b_i = x_i + c_i$ be the beginning of the green at signal S_i. Then E, the end of the green band on the artery is given by

$$E = \min \{e_i + TT(i,n) \mid i = 1, 2, \ldots, N\}$$

$= \min\{x_i+c_i+TT(i,n) \mid i = 1, 2, \ldots, N\}$ and B, the beginning of the green band is given by

$$B = \max\{b_i+TT(i,n) \mid i = 1,2, \ldots, N\}$$
$$= \max \{x_i+k_i+TT(i,n) \mid i = 1,2, \ldots, N\}.$$

These values are used to see that the bandwidth on the artery is expressible as a function of the N variables, $x_1, x_2, \ldots, x_N$. Thus it has been demonstrated that the following claim is true.

CLAIM 1: The bandwidth on an N signal artery is expressible as

$$R(x_1,X_2,\ldots,X_N) = \min \{x_i+k_i+TT(i,n) \} - \max \{x_i+c_i+TT(i,n)$$

where x_i is the offset at signal Si,

k_i and c_i are constants,

$TT(i,N)$ is the travel time from signal S_i to signal S_N, $i = 1,2,3,\ldots,n$.

The lemma below shows that a number expressible as a minimum minus a maximum can be found in steps by successively adding numbers to the set of numbers that the minimum and maximum are taken over. Thus Claim 1 above and Lemma 1 below will show that the bandwidth on an artery can be found by first finding the maximum bandwidth between the first and last signals, then successively adding constraints introduced by the middle signals one at a time. This method of computing the bandwidth allows the computation of the maximum sum of inbound and outbound bandwidths quite simply by finding the maximum value of a sum of two piecewise linear functions as illustrated in the two-signal case.

LEMMA 1: Let $A = \{a_1, a_2, \ldots, a_n\}$, $B = \{b_1, b_2, \ldots, b_n\}$, $A' \subseteq A$, and $B' \subseteq B$, then

$$\min A - \max B \leq \min A' - \max B' .$$

proof: Since min A is a minimum over more numbers than min A′ it follows that $\min A \leq \min A'$. Likewise $\max B \geq \max B'$ (since the maximum is taken over more numbers). Thus we have $\min A \leq \min A'$ and $\max B \geq \max B'$. Multiplying both sides of the last inequality by -1 gives $-\max B \leq -\max B'$. Finally, adding the two inequalities gives the required inequality,

$$\min A - \max B \leq \min A' - \max B'.$$

COMPUTATION OF THE GREEN BAND ON AN ARTERIAL HIGHWAY

Using claim 1 and lemma 1 above the bandwidth on an artery can be calculated by computing the maximum value of N-1 piecewise linear functions sequentially. Denote

$$\min \{x_1+k_1+TT(1,N), x_N+k_N+TT(1,N)\}$$
$$- \max\{x_1+c_1+TT(1,N), x_N+c_N+TT(1,N)\}$$

by $R(x_1,x_N)$ and in general let $R(x_1,x_2,\ldots,xi,x_N)$ denote

$$\min \{x_1+k_1+TT(1,N), \ldots, x_i+k_i+TT(i,N), x_N+k_N+TT(1,N)\}$$
$$- \max\{x_1+c_1+TT(1,N), \ldots, x_i+c_i+TT(i,N), x_N+c_N+TT(1,N)\}.$$

Then $R(x_1,x_N)$ is the maximum bandwidth between signals 1 and N, ignoring the remaining signals. $R(x_1,x_2,\ldots,x_i,x_N)$ is the bandwidth on the artery from signals 1 to i. Lemma 1 shows that

$$\begin{aligned} R(x_1,x_2,\ldots,x_N) &\leq R(x_1,x_2,\ldots,x_{N-2},x_N\} \\ &\leq R(x_1,x_2,\ldots,x_{N-3},x_N) \\ &\leq R(x_1,x_2,\ldots,x_{N-4},x_N) \\ &\leq \ldots \leq \ldots \leq R(x_1,x_2,x_N) \leq R(x_1,x_N). \end{aligned}$$

The value of $R(x_1,x_N)$ can be computed as in chapter two as the maximum of a piecewise linear function of one variable. Let

$$\mathrm{curmin}_1 = \min\{x_1+k_1+TT(1,N), x_N+k_N+TT(1,N)\}$$

and $\mathrm{curmax}_1 = \max\{x_1+c_1+TT(1,N), x_N+c_N+TT(1,N)\}$.

Then the bandwidth between signals 1 and N is given by

$R(x_1,x_N) = \mathrm{curmin}_1 - \mathrm{curmax}_1$. Also,

$$\begin{aligned} R(x_1,x_2,x_N) &= \min\{x_1+k_1+TT(1,N), x_2+k_2+TT(2,N), x_N+k_N+TT(1,N)\} \\ &\quad - \max\{x_1+c_1+TT(1,N), x_2+c_2+TT(1,N)\} \\ &= \min\{x_2+k_2+TT(2,N), \mathrm{curmin}_1\} \\ &\quad - \max\{x_2+c_2+TT(2,N), \mathrm{curmax}_1\}. \end{aligned}$$

Similarly, $R(x_1,x_2,\ldots,x_i,x_N) = \min\{x_i+k_i+TT(i,N), \mathrm{curmin}_{i-1}\} - \max\{x_i+c_i+TT(i,N), \mathrm{curmax}_{i-1}\}$

where

$\mathrm{curmin}_i = \min\{x_1+k_1+TT(1,N), \ldots, x_i+k_i+TT(i,N), x_N+k_N+TT(1,N)\}$

and

$\mathrm{curmax}_i = \max\{x_1+c_1+TT(1,N), \ldots, x_i+c_i+TT(i,N), x_N+c_N+TT(1,N)\}$.

After N-1 iterations of this process, the bandwidth on the artery is found to be

$$\begin{aligned} R(x_1,x_2,\ldots,x_N) &= \min\{x_{N-1}+k_{N-1}+TT(N-1,N), \mathrm{curmin}_{N-2}\} \\ &\quad - \max\{x_{N-1}+c_{N-1}+TT(N-1,N), \mathrm{curmax}_{N-2}\} \\ &= \mathrm{curmin}_{N-1} - \mathrm{curmax}_{N-1}. \end{aligned}$$

Thus to compute the bandwidth on the artery, first compute curmin_1 and curmax_1, then compute curmin_i and curmax_i for i = 2 to N-1 sequentially as maximum values of piecewise linear functions as in chapter 2. The bandwidth is then $\text{curmin}_{N-1} - \text{curmax}_{N-1}$.

SUMMARY OF ALGORITHM OPTIMIZING AN N SIGNAL ARTERY

Step 1. Find the phase sequence combination(s) and corresponding offsets which yield the maximum sum of inbound and outbound bands between signals 1 and N. This is done by using the two signal arterial algorithm presented in chapter 2.

Do the following for each phase sequence at signal i, i= 2 to N-1.

Step 2. Fix the offset and phase sequences at signals 1 and N. Then compute

$$\min\{e_1+TT(1,N),\ldots, e_i+TT(i,N),\ldots, e_N\}$$
$$- \max\{b_1+TT(1,N),\ldots, b_i+TT(i,N),\ldots, b_N\}$$

for each of the four phase sequence combinations.

[Note that the only variable in the above expression is x_i, the offset at signal S_i].

Step 3. Select the maximum of the four values computed in step 2. Fix the offset , x_i, and the phase sequence at signal i to be (one of) the phase sequences which yields the maximum sum of inbound and outbound bandwidths.

Example Arterial Highways

To see how the algorithm developed in this chapter works, two example arterial highways are optimized below. Appendix B is a listing of a computer program that was written and used as an aid in applying the algorithms of this research to the examples.

The first arterial highway to be optimized consists of three signalized intersections along an east-west corridor. The distances between intersections and travel times are shown in table 3.1 and the green requirement per movement is shown in table 3.2.

TABLE 3.1: DISTANCES BETWEEN INTERSECTIONS AND TRAVEL TIMES ON EXAMPLE 1

LINK	LINK LENGTH	TRAVEL TIME
1 to 2	1050 ft	25 secs
2 to 1	1050 ft	25 secs
2 to 3	975 ft	23 sec
3 to 2	1000 ft	20 sec

TABLE 3.2: GREEN TIME REQUIREMENTS BY MOVEMENT FOR EXAMPLE 1

MOVEMENT	SIGNAL 1	SIGNAL 2	SIGNAL 3
Outbound through	30	20	35
Outbound left	15	25	10
Inbound through	25	30	25
Inbound left	10	15	10

Solution to Example Arterial Highway 1

The first step in selecting optimal phase sequence combinations at each signal of the arterial highway is to determine the green times per phase for each of the four possible left turn patterns. These assignments are shown below in table 3.3. Using the green time per phase, the

TABLE 3.3: GREEN TIME ASSIGNMENTS BY MOVEMENT FOR EXAMPLE 1

SIGNAL	PHASE SEQUENCE	O	I	T	L
1	Lead-LAG	15	10	35	0
	LAG-Lead	15	10	35	0
	LAG-LAG	5	0	45	0
	Lead-Lead	5	0	45	0
2	Lead-LAG	25	10	15	0
	LAG-Lead	25	10	15	0
	LAG-LAG	10	0	25	15
	Lead-Lead	10	0	25	15
3	Lead-LAG	10	10	30	0
	LAG-Lead	10	10	30	0
	LAG-LAG	0	0	40	10
	Lead-Lead	0	0	40	10

beginning and end ofgreen calculations are made at each intersection for each phase sequence.

After determining the green time per phase, the next step is to define the bandwidth functions and finally to calculate the maximum sum of the inbound and outbound bands between signals 1 and 3. The maximum sum for each combination of phase sequence combinations is shown in table 3.5 along with the offsets which give the maximum sum

TABLE 3.4: MAXIMUM SUM OF BANDWIDTHS BETWEEN SIGNALS 1 AND 3 FOR EXAMPLE 1

PHASE SEQUENCE	MAXIMUM SUM OF BANDS	OFFSET(S) SIGNAL 2
LAG-LAG/LAG-LAG	0	0
LAG-LAG/Lead-Lead	0	0
LAG-LAG/Lead-LAG	0	0
LAG-LAG/LAG-Lead	73	[48,55]
Lead-Lead/LAG-LAG	78	[68,70]
Lead-Lead/Lead-Lead	78	[58,60]
Lead-Lead/Lead-LAG	80	[60,65]
Lead-Lead/LAG-Lead	68	[58,70]
Lead-LAG/LAG-LAG	68	[58,70]
Lead-LAG/Lead-Lead	68	[58,70]
Lead-LAG/Lead-LAG	78	[58,60]
Lead-LAG/LAG-Lead	58	[48,70]
LAG-Lead/LAG-LAG	80	[58,60]
LAG-Lead/Lead-Lead	45	[48,50]
LAG-Lead/Lead-LAG	60	[45,48]
LAG-Lead/LAG-Lead	60	0

Table 3.4 shows that two phase sequence combinations yield the maximum possible sum of bands,lead-lead /lead-lag, and lag-lead/lag-lag. Signal 2, the middle signal, is introduce after assigning phase sequence lag-lead and offset 0 at signal 1 and phase sequence lag-lag and offset 58 at

signal 3. Table 3.5 shows the calculated maximum sum of inbound and outbound bands for each of the four phase sequence combinations at signal 2. Phase sequence lead-lag yields a maximum sum of bands of 47. Thus, the solution to the example is found and is illustrated in table 3.6. Figure 3.1 shows the corresponding time space diagram.

TABLE 3.5: MAXIMUM SUM OF GREEN BANDS BY PHASE SEQUENCE AT SIGNAL 2 OF EXAMPLE 1

PHASE SEQUENCE	OPTIMAL OFFSET	MAXIMUM SUM OF BANDS
Lag-Lag	[78,93]	25
Lead-Lead	[20,25]	35
Lag-Lead	[40,53]	47
Lead-Lag	[78,93]	25

TABLE 3.6: OPTIMAL SOLUTION TO EXAMPLE 1

SIGNAL	PHASE- SEQUENCE	OFFSET
1	Lag-Lead	0
2	Lead-Lag	40
3	Lag-Lag	58
INBOUND BAND 12 SECS OUTBOUND BAND 35 SECS		

Example Arterial Highway #2

The next arterial example is what might be considered a worst case of spacing example. This arterial highway consists of five signalized intersections along an east-west corridor.

The distances between intersections and travel times are shown in table 3.7 and the green requirement per movement is shown in table 3.8.

TABLE 3.7: DISTANCES BETWEEN INTERSECTIONS AND TRAVEL TRAVEL TIMES ON EXAMPLE 2

LINK	LINK LENGTH	TRAVEL TIME
1 to 2	1500 ft	34 secs
2 to 1	1500 ft	34 secs
2 to 3	660 ft	15 secs
3 to 2	660 ft	15 secs
3 to 4	660 ft	15 secs
4 to 3	660 ft	15 secs
4 to 5	1500 ft	34 secs
5 to 4	1500 ft	34 secs

TABLE 3.8: GREEN TIME REQUIREMENTS BY MOVEMENT FOR EXAMPLE 2

MOVEMENT	SIG 1	SIG 2	SIG 3	SIG 4	SIG 5
Outbound through	20	20	20	20	20
Outbound left	10	10	10	10	10
Inbound through	20	20	20	20	20
Inbound left	10	10	10	10	10

Solution to Example Arterial Highway 2

First, the green times per phase for each of the four possible left turn phase sequences are computed. These assignments are shown below in table 3.9. Using the green time per phase, the beginning and end of green calculations are made at each intersection for each phase sequence.

TABLE 3.9: GREEN TIME ASSIGNMENTS BY PHASE SEQUENCE FOR EXAMPLE 2

SIGNAL	PHASE SEQUENCE	O	I	T	L
1	Lead-Lag	10	10	20	0
	Lag-Lead	10	10	20	0
	Lag-Lag	0	0	30	10
	Lead-Lead	0	0	30	10
2	Lead-Lag	10	10	20	0
	Lag-Lead	10	10	20	0
	Lag-Lag	0	0	30	10
	Lead-Lead	0	0	30	10
3	Lead-Lag	10	10	20	0
	Lag-Lead	10	10	20	0
	Lag-Lag	0	0	30	10
	Lead-Lead	0	0	30	10
4	Lead-Lag	10	10	20	0
	Lag-Lead	10	10	20	0
	Lag-Lag	0	0	30	10
	Lead-Lead	0	0	30	10
5	Lead-Lag	10	10	20	0
	Lag-Lead	10	10	20	0
	Lag-Lag	0	0	30	10
	Lead-Lead	0	0	30	10

After determining the green time per phase, the next step is to define the bandwidth functions and finally to calculate the maximum sum of the inbound and outbound bands between signals 1 and 3. The maximum sum for each combination of phase sequence combinations is shown in table 3.10 along with the offsets which give the maximum sum. Table 3.10 shows that phase sequence lead-lag/lag-lead yields the maximum possible sum of bands of 56. Signal 2 is considered next after assigning phase sequence lead-lag and offset 0 at signal 1 and phase sequence lag-lead and offset 28 at signal 5.

Table 3.11 shows the calculated maximum sum of inbound and outbound bands for each of the four phase sequence combinations at signal 2. Phase sequence lead-lead yields a maximum sum of bands of 56. Next signals 3 and 4 are introduced into the optimization process. Eventually, the solution to the example is found and is illustrated in table 3.12.

TABLE 3.10: MAXIMUM SUM OF BANDWIDTHS BETWEEN SIGNALS 1 AND 5 FOR EXAMPLE 2

	MAXIMUM SUM	OFFSET(S)
Lag-Lag/Lag-Lag	44	[22,38]
Lag-Lag/Lead-Lead	44	[12,20]
Lag-Lag/Lead-Lag	34	[12,38]
Lag-Lag/Lag-Lead	54	[22,28]
Lead-Lead/Lag-Lag	44	[32,48]
Lead-Lead/Lead-Lead	44	[22,38]
Lead-Lead/Lead-Lag	34	[22,48]
Lead-Lead/Lag-Lead	54	[32,38]
Lead-Lag/Lag-Lag	54	[32,38]
Lead-Lag/Lead-Lead	54	[32,38]
Lead-Lag/Lead-Lag	44	[22,38]
Lead-Lag/Lag-Lead	56	[28,32]
Lag-Lead/Lag-Lag	34	[22,48]
Lag-Lead/Lead-Lead	30	12
Lag-Lead/Lead-Lag	30	12
Lag-Lead/Lag-Lead	44	[22,38]

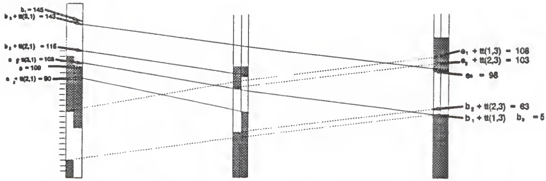
TABLE 3.11: MAXIMUM SUM OF GREEN BANDS BY PHASE SEQUENCE AT SIGNAL 2 OF EXAMPLE 2

PHASE SEQUENCE	OPTIMAL OFFSET	MAXIMUM SUM OF BANDS
Lag-Lag	34	56
Lead-Lead	24	56
Lag-Lead	[26,34]	48
Lead-Lag	[24,32]	48

TABLE 3.12: OPTIMAL SOLUTION TO EXAMPLE 2

SIGNAL	PHASE-SEQUENCE	OFFSET
1	Lead-Lag	0
2	Lead-Lead	40
3	Lag-Lead	58
4	Lead-Lag	4
5	Lag-Lead	28

INBOUND BAND 8 secs
OUTBOUND BAND 30 secs



OUTBOUND BANDWIDTH = MIN{ 108, 103, 98} - MAX{ 58, 58, 63 }
= 98 - 63 = 35

INBOUND BANDWITH = MIN{{ 145,143,115 } - MAX{ 103, 90, 100 } -
= 115 - 103 = 12

FIGURE 3.1: TIME SPACE DIAGRAM OF SOLUTION TO EXAMPLE 1

CHAPTER FOUR

APPLICATIONS TO CLOSED NETWORKS

The phase sequence optimization problem is now extended to the simple closed loop consisting of four arteries intersecting at four signals.

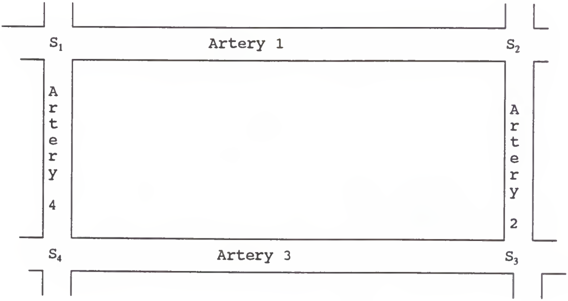


FIGURE 4.1: SIMPLE CLOSED LOOP

Optimization of each artery of the loop using the procedures of chapter 2 above yields offsets and phase sequence patterns at each intersection which result in the maximum sum of bands on each artery of the loop. The optimization of each artery yields the following optimal assignments.

Let x_i represent the offset at signal S_i, EWP_i represent the east-west phase pattern at signal S_i, and NSP_i the north south phase pattern at signal S_i.

Artery Optimized	Phase Pattern Selected	Offset Selected
Artery 1	EWP_1 and EWP_2	x_2
Artery 2	NSP_2 and NSP_3	x_3
Artery 3	EWP_3 and EWP_4	x_4
Artery 4	NSP_1 and NSP_4	x_1

The values selected for x_i must satisfy the following equations and inequalities.

$$\begin{aligned} x_2 &= x_1 + y_2, & a_2 \le y_2 \le b_2 \\ x_3 &= x_2 + y_3, & a_3 \le y_3 \le b_3 \qquad \text{Conditions 4.1} \\ x_4 &= x_3 + y_4, & a_4 \le y_4 \le b_4 \\ x_1 &= x_4 + y_1, & a_1 \le y_1 \le b_1 \end{aligned}$$

[Values for the a_i's and b_i's represent the endpoints of the interval on which each maximum sum is attained]

The network closure constraint requires that the offsets are selected so that the sum of the offsets around the loop should be an integral multiple of the cycle length. Thus an additional requirement on the x_i's is that $\Sigma\, x_i = mCL$ for some integer m, where CL represents the systemwide cycle length.

LEMMA 2: There exists x_i, $i = 1, 2, 3, 4$, satisfying conditions 4.1 and such that $\Sigma\, x_i = mCL$ if and only

if $\Sigma\, a_i \leq MCL \leq \Sigma\, b_i$.

proof: If $\Sigma\, a_i \leq MCL \leq \Sigma\, b_i$ for some M, then consider

case 1. If $a_2 + MCL - \Sigma\, a_i \leq b_2$

Choose

$$y_2 = a_2 + MCL - \Sigma\, a_i$$

$$y_3 = a_3$$

$$y_4 = a_4$$

$$y_1 = a_1$$

then $a_i \leq y_i \leq b_i$ for each i,

and if $x_1 = [\ 2MCL + 3a_1 + a_3 + 2a_4]/4$

$x_2 = [\ 2MCL - a_1 - 3a_3 - 2a_4]/4$

$x_3 = [\ 2MCL - a_1 + a_3 - 2a_4]/4$

$x_4 = [\ 2MCL - a_1 + a_3 + 2a_4]/4$

then $\Sigma\, x_i = 2{*}M{*}CL$.

Thus if $m = 2M$ then $\Sigma\, x_i = mCL$.

case 2: If $a_2 + MCL - \Sigma\, a_i > b_2$, then choose

$y_2 = b_2$ and let $k_2 = a_2 + MCL - \Sigma\, a_i - b_2$.

case 2A. If $a_3 + k_2 \leq b_3$ then choose

$$y_3 = a_3 + k_2$$

$$= MCL - a_1 - a_4 - b_2$$

$$y_4 = a_4$$

$$y_1 = a_1$$

and $x_1 = [2a_1 + a_4 - b_2]/4$

$x_2 = [2a_1 + a_4 = 3b_2]/4$

$x_3 = [4MCL - 2a_1 - 3a_4 - b_2]/4$

$x_4 = [4MCL - 2a_1 + a_4 - b_2]/4$

then $\Sigma x_i = 2MCL$.

case 2B. If $a_3 + k_2 > b_3$ then choose $y_3 = b_3$ and

let $k_3 = b_3 - a_3 - k_2 = b_3 - MCL + a_1 + a_4 + b_2$

case 2Bi . If $a_4 + k_3 \leq b_4$ then choose

$y_4 = a_4 + k_3$

$y_1 = a_1$

and let $x_1 = [3MCL - 4b_2 - 3b_3 - 2a_4 - a_1]/4$

$x_2 = [3MCL - 3b_3 - 2a_4 - a_1]/4$

$x_3 = [3MCL + b_3 - 2a_4 - a_1]/4$

$x_4 = [MCL + 5b_3 + 6a_4 + 3a_1 + 4b_2]/4$

then $\Sigma x_i = 2MCL$

case 2Bii. If $a_4 + k_3 > b_4$ then choose

$y_4 = b_4$

$y_1 = a_1 + b_4 - b_3 - b_2 + MCL - a_1 - 2a_4$

and let $x_1 = [2MCL - 3b_2 - 2b_3 - b_4]/4$

$x_2 = [2MCL + b_2 - 2b_3 - b_4]/4$

$x_3 = [2MCL + b_2 + 2b_3 - b_4]/4$

$x_4 = [2MCL + b_2 + 2b_3 + 3b_4]/4$

then $\Sigma x_i = 2MCL$.

Therefore in each case there exist x_i, i = 1,2,3,4 which satisfy the conditions required.

Lemma 2 shows that a solution which is optimal on all four arteries of the loop exists if and only if a multiple of the cycle length lies between the sum of the a_i's and the sum of the b_i's. The following example illustrates such a case.

EXAMPLE 3 A SIMPLE CLOSED LOOP

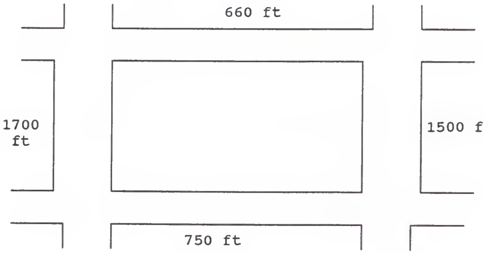


FIGURE 4.2: ILLUSTRATION OF EXAMPLE 3

TABLE 4.1: GREEN REQUIREMENT BY MOVEMENT FOR EXAMPLE 3

SIGNAL	NBL	NBT	SBL	SBT	EBL	EBT	WBL	WBT
1	10	15	0	20	10	20	10	20
2	0	15	0	15	10	20	10	20
3	0	15	0	15	10	20	10	20
4	10	15	0	20	10	20	10	20

TABLE 4.2: LINK TRAVEL TIMES FOR EXAMPLE 3

LINK	TT	LINK	TT
1 TO 2	15	3 TO 4	17
2 TO 1	15	4 TO 3	17
2 TO 3	34	4 TO 1	38
3 TO 2	34	1 TO 4	38

The algorithm developed in chapter two yields the maximum sum of inbound and outbound bands on the four arteries of the loop. Table 4.3 below shows the results of applying the algorithm.

TABLE 4.3: MAXIMUM SUM OF BANDWIDTHS BY ARTERY OF EXAMPLE 3

ARTERY	MAXIMUM SUM OF BANDS	OPTIMAL PHASE PATTERN	OFFSET INTERVAL
1	40	Lag-Lead/Lag-Lag	$25 \leq y_2 \leq 45$
2	30	Lag-Lead/Lag-Lead	$y_3 = 34$
3	44	Lag-Lead/Lag-Lag	$27 \leq y_4 \leq 43$
4	24	Lead-Lag/Lag-Lead	$32 \leq y_1 \leq 38$

Since 25 + 34 + 27 + 32 = 118 and 45 + 34 +43 + 38 = 160, it follows that 2*CL = 120 lies between Σa_i and Σb_i. The proof of claim 3 illustrates how the offsets, x_i, i = 1,2,3, 4 can be chosen to satisfy conditions 4.1. Choose y_2= 25+120-118 = 27, y_3= 34, y_4= 27, and y_1= 32. Then the offsets x_i, at each signal are found to be x_1= 1, x_2= 28, x_3= 2, and x_4 = 29 ($\Sigma x_i = 60$).

Thus a solution which yields the maximum sum of inbound and outbound bands on each artery of the network is found and summarized in table 4.4 below.

TABLE 4.4: OPTIMAL SOLUTION TO EXAMPLE 3

SIGNAL	EW PHASING	NS PHASING	OFFSET
1	Lag-Lead	Lag-Lead	1
2	Lag-Lag	Lag-Lead	28
3	Lag-Lead	Lag-Lead	2
4	Lag-Lag	Lead-Lag	29

If there does not exist M so that MCL lies between Σa_i and Σb_i then at least one artery of the loop will attain less than its absolute maximal sum of bands. The goal in optimization then is to widen the intervals in a way that will include a multiple of the cycle length and still obtain a sum of bands which is as close as possible to the maximum. The appropriate compromise to make in this case is a matter of judgement. One can make the decision based on

1. Whether one direction is favored over others,
2. Whether one artery is less important than others,
3. A desire to maintain a minimum sum of bands throughout the loop, or
4. Some other engineering decision.

The approach taken by this author is to sacrifice one artery of the loop. That is, if the engineer's decision is to allow one artery of the loop to attain a suboptimal sum of bands in order to satisfy the network closure constraint (e.g. the artery with the lowest maximum sum of bands), then a solution yielding the maximum sum of bands on the remaining three arteries of the loop is possible. Consider the set of constraining equations 4.1, and suppose without loss of generality that artery four is to be sacrificed. Then the new set of constraints are

$x_2 = x_1 + y_2$, $a_2 \leq y_2 \leq b_2$

$x_3 = x_2 + y_3$, $a_3 \leq y_3 \leq b_3$ Constraints 4.2

$x_4 = x_3 + y_4$, $a_4 \leq y_4 \leq b_4$

$x_1 = x_4 + y_1$, $0 \leq y_1 \leq CL$

and $\Sigma x_i = MCL$ for some M.

Claim 4: There exist a multiple of CL between

$a_2 + a_3 + a_4$ and $CL + b_2 + b_3 + b_4$,

Proof: Clearly

$$a_2 + a_3 + a_4 \leq CL + b_2 + b_3 + b_4 < 4 * CL.$$

Case 1. Now suppose $4*CL < a_2 + a_3 + a_4$, then $4*CL < a_2 + a_3 + a_4 \leq CL + b_2 + b_3 + b_4 \leq 4* CL$. This implies that we have equality throughout the inequality. That is,

$$a_2 + a_3 + a_4 = CL + b_2 + b_3 + b_4 = 4* CL .$$

Thus 4* CL lies in the interval.

Case 2. If $3*CL < a_2 + a_3 + a_4$, then

$3*CL < a_2 + a_3 + a_4 \leq b_2 + b_3 + b_4 \leq 3* CL$.

This also implies equality throughout and thus,

$a_2 + a_3 + a_4 = CL + b_2 + b_3 + b_4 = 3* CL$.

Thus 3* CL lies in the interval.

Case 3. If $2*CL < a_2 + a_3 + a_{4,}$ then $2*CL < a_2 + a_3 + a_4 \leq b_2 + b_3 + b_4 \leq 3*CL$. But , if 3*CL is between $a_2 + a_3 + a_4$ and $b_2 + b_3 + b_4$ then case 2 above applies. Meaning that 3*CL lies in the interval. Otherwise 2*CL lies in the interval.

Case 4. If $CL < a_2 + a_3 + a_4$, then

$$CL < a_2 + a_3 + a_4 \leq 3*CL.$$

In this case either 2*CL or CL lie in the interval.

Example 4.2 below illustrates how the proof of Claim 4.2 can be used to "optimize" a loop where the absolute maximum sum of bands on all four arteries is not possible.

EXAMPLE 4: SIMPLE CLOSED LOOP - SUBOPTIMAL SOLUTION ON ONE ARTERY REQUIRED

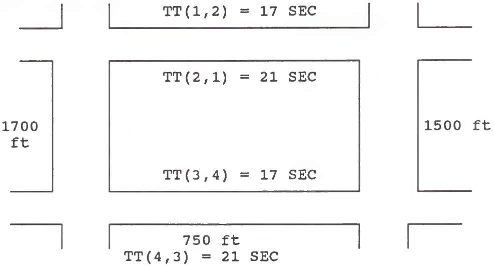


FIGURE 4.3: ILLUSTRATION OF EXAMPLE 4

TABLE 4.5: GREEN REQUIREMENT BY MOVEMENT FOR EXAMPLE 4

SIGNAL	NBL	NBT	SBL	SBT	EBL	EBT	WBL	WBT
1	10	25	25	10	10	25	25	10
2	25	10	10	25	25	10	10	25
3	10	30	10	30	10	30	10	30
4	10	30	10	30	10	30	10	30

TABLE 4.6: LINK TRAVEL TIMES EXAMPLE 4

LINK	TT	LINK	TT
1 TO 2	17	3 TO 4	17
2 TO 1	21	4 TO 3	21
2 TO 3	17	4 TO 1	17
3 TO 2	21	1 TO 4	21

The algorithm of chapter two again yields the optimum solution on each artery of the loop. The optimization results are shown in table 4.7 below.

TABLE 4.7: MAXIMUM SUM OF BANDWIDTHS BY ARTERY OF EXAMPLE 4

Maximum Sum of Bands	Optimal Phase Pattern	offset interval
38	lead-lag/lead-lag	$27 \leq y_1 \leq 29$
38	lead-lag/lead-lag	$27 \leq y_2 \leq 29$
38	lead-lag/lead-lag	$27 \leq y_3 \leq 29$
38	lead-lag/lead-lag	$27 \leq y_4 \leq 29$

Since 27 + 27 + 27 + 27 = 108 and 29 + 29 +29 + 29 = 116, it is clear that no multiple of the cycle length (60 secs) lies between 108 and 116. Thus we choose one artery to be "sacrificed." Choosing to sacrifice artery 4 the offsets x_i, at each signal are found to be $x_1 = 13$, $x_2 = 42$, $x_3 = 19$, and $x_4 = 46$. ($x_1 + x_2 + x_3 + x_4 = 120$).

Thus a solution which yields the maximum sum of inbound and outbound bands on three of the four arteries has been found and is summarized in the table 4.8 below.

TABLE 4.8: OPTIMAL SOLUTION TO EXAMPLE 4

SIGNAL	EW PHASING	NS PHASING	OFFSET	SUM OF BANDS
1	Lead-Lag	Lead-Lag	13	38
2	Lead-Lag	Lead-lag	42	23
3	Lead-Lag	Lead-Lag	19	38
4	Lead-Lag	Lead-Lag	46	23

Example 4.3: Daytona Beach Network

To show how the procedure described above can be used in conjunction with existing signal timing tools, the twelve signal grid network used in the TRANSYT-7F user's manual[29] as an example will now be optimized. The network consists of three east west arterial highways and four north south arterial highways. Figure 4.4 is an illustration of this network. TRANSYT-7F only optimizes cycle length, splits and offsets. The procedure above will be used to obtain an even better solution for the network by finding phase sequence patterns at each intersection which using the given splits, cycle length and offsets will yield the greatest sum of inbound and outbound bandwidths on the arteries of the network. Table 4.9 shows each optimal value as determined by TRANSYT-7F. Using these values the procedure above yields optimal phase sequence patterns shown in table 4.10.

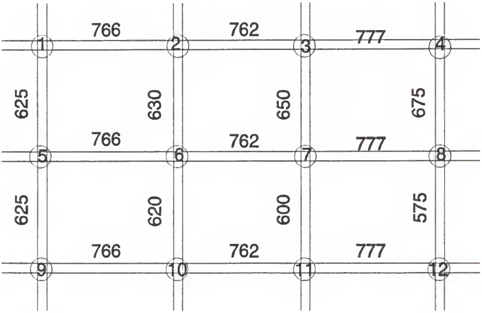


FIGURE 4.4: ILLUSTRATION OF EXAMPLE 5 DAYTONA BEACH NETWORK

Table 4.9 TRANSYT-7F OPTIMIZATION RESULTS ON EXAMPLE 5 DAYTONA BEACH NETWORK

SIGNAL	SPLIT EW/NS	OFFSET	SIGNAL	SPLIT EW/NS	OFFSET
1	79/41	0	7	34/86	119
2	102/18	107	8	68/52	96
3	62/58	42	9	66/54	4
4	48/72	41	10	80/40	83
5	34/86	115	11	78/42	19
6	82/38	98	12	82/38	51

TABLE 4.10: OPTIMAL PHASE SEQUENCES BY SIGNAL FOR DAYTONA BEACH NETWORK

SIGNAL	EAST-WEST SUBSEQUENCE	NORTH-SOUTH SUBSEQUENCE
1	Lead-Lead	Lag-Lead (ANY)
2	Lead-Lag	Lag-Lead (ANY)
3	Lead-Lag	Lag-Lead (ANY)
4	Lag-Lead	Lag-Lag
5	Lag-Lead	Lag-Lead (ANY)
6	Lag-Lead (ANY)	Lag-Lead (ANY)
7	Lag-Lead (ANY)	Lag-Lead (ANY)
8	Lag-Lead	Lag-Lead (ANY)
9	Lag-Lead	Lag-Lead (ANY)
10	Lag-Lead (ANY)	Lag-Lead (ANY)
12	Lag-Lead	Lead-Lag

ANY - INDICATES THAT NO PHASE SEQUENCE IS BETTER THAN THE OTHERS

Table 4.11 shows a comparison of several performance indexes of the TRANSYT-7F solution and the solution obtained using the procedure of this research. The comparison shows that the solution obtained using the new procedure improves each measure of performance on the network and substantially improves (10.62% increase) the overall sum of bandwidths.

TABLE 4.11: COMPARISON OF PERFORMANCE MEASURES ON EXAMPLE 5 DAYTONA BEACH

PERFORMANCE INDEX	TRANSYT-7F SOLUTION BANDS NOT OPTIMIZED	SOLUTION WITH BANDS OPTIMIZED
TRANSYT-7F PI	4113.21	4100.22
FUEL CONSUMPTION	9119.47	9079.71
SUM OF BANDWIDTHS	271.20	300.00

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The previous chapters have shown that the optimization of the signal timing components in networks of multiphase signals can be accomplished without resulting to trial and error methods. A closed form function of n variables has been defined whose dependent variable is the sum of inbound and outbound bandwidths on an artery and whose maximum value is the maximum sum of inbound and outbound bandwidths on the arterial highway. The functions defined can be used to select phase sequence combinations which yield a maximal sum of inbound and outbound bandwidths on the arteries of a network of multiphase signals.

The definition of the bandwidth function requires that the cycle length and green allocation at each intersection be given. The values are then used to define the bandwidth on each arterial highway as a function whose independent variables are the offsets at the signals of the network. An analysis of the function showed that the maximum value of the function can be found by evaluating a few simple algebraic expressions. The maximum sum of several of the bandwidth functions can be obtained simply and quickly (less than one second per calculation) on a personal computer. Using these maximum values a method of selecting optimal phase sequence combinations at each signal of a network of multiphase signals has been demonstrated. The method selects the combination of phase sequence combinations which give the maximum sum of green bands. Thus, the procedure developed is a practical tool which can be used in conjunction with existing programs to completely optimize networks of multiphase signals.

The algorithm not only selects optimal phase sequences for arterial highways but also provides a range of offsets associated with the phase sequences. The network solution selects optimal phase sequences for each approach of the network.

Drawing on the fact that cycle length, splits, and offset optimization algorithms are readily available to the traffic engineer a final step in the network optimization process has been developed. The selection procedure developed has the advantage of requiring very little computer resources and therefore would be a cost effective addition to the network optimization process. The algorithm can be included as an additional step in network optimization using other optimization tools.

Recommendations

Since the technique developed finds all phase sequence combinations which yield a maximum sum of bands, it is suggested that further research be done on ranking the four phase sequence combinations and on selecting one sequence among many which offer the same or nearly the same sum of bands.

The procedure as demonstrated considers all four of the phase sequence as viable subsequences at each signal of the network. It is also recommended that further investigation on the possibility of excluding one or more of the phase sequences at some signals of the network be carried out.

The fact that the procedure developed can be used to select offsets on an arterial highway suggests that further research on how to use the bandwidth function to select offsets in a network is also needed.

A comparison of the results of the procedure developed above and the results obtained using existing optimization MAXBAND-86[19] should be undertaken. This would require significant computer resources to accommodate MAXBAND-86 and that the procedure be more efficiently programmed for computer implementation.

Finally, it is recommended that the algorithms developed during this research be programmed and then used in conjunction with existing programs, such as TRANSYT, which optimize all of the other components of signal timing.

If this algorithm is used in the recommended manner, networks can be completely optimized using less computer time and thereby spending fewer dollars.

APPENDIX A

DEFINITIONS OF BANDWIDTH FUNCTIONS

For each of the sixteen combinations of phase sequences the inbound and outbound bandwidth functions are of the general form

$$F_i(X) = \begin{cases} X + C_i & X < A_i \\ A_i + C_i & A_i \leq X \leq B_i \\ A_i + C_i + B_i - X & X > B_i. \end{cases}$$

Definitions of A_i, B_i, and C_i for each of the sixteen phase sequence combinations are listed below. Definitions are identified using the phase sequence at the first signal followed by the phase sequence at the second signal.

The following symbols are used in defining the functions:

T_i = Number of seconds of green allotted to the through movement

L_i = Number of seconds of green allotted to the left turn movement

O_i = Number of seconds of green allotted to the outbound through and outbound left turn movement

I_i = Number of seconds of green allotted to the inbound through and inbound left turn movement .

1. LAG-LAG/LAG-LAG SEQUENCE COMBINATION

$A_1 = \min\{TT(1,2),\ TT(1,2)+O_1+T_1-O_2-T_2\}$

$B_1 = \max\{TT(1,2),\ TT(1,2)+O_1+T_1-O_2-T_2\}$

$C_1 = T_2+O_2-TT(1,2)$

1. LAG-LAG/LAG-LAG SEQUENCE COMBINATION CONTINUED

$A_2 = \min \{TT(2,1),\ TT(2,1)+T_1+I_1-T_2-I_2\}$

$B_2 = \max \{TT(2,1),\ TT(2,1)+T_1+I_1-T_2-I_2\}$

$C_2 = T_2+I_2-TT(2,1)$

2. LAG-LAG/LEAD-LEAD SEQUENCE COMBINATION

$A_1 = \min \{TT(1,2)-I_2-L_2,\ TT(1,2)-I_2-L_2+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2)-I_2-L_2,\ TT(1,2)-I_2-L_2+O_1+T_1-O_2-T_2\}$

$C_1 = I_2+L_2+O_2+T_2-TT(1,2)$

$A_2 = \min \{TT(2,1)-L_2-O_2,\ TT(2,1)-L_2-O_2+T_1+I_1-T_2-I_2\}$

$B_2 = \max \{TT(2,1)-L_2-O_2,\ TT(2,1)-L_2-O_2+T_1+I_1-T_2-I_2\}$

$C_2 = I_2+L_2+O_2+T_2-TT(2,1)$

3. LAG-LAG/LEAD-LAG SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2),\ TT(1,2)+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2),\ TT(1,2)+O_1+T_1-O_2-T_2\}$

$C_1 = O_2+T_2-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1)-O_2,\ TT(2,1)-O_2+T_1+I_1-T_2-I_2$

$B_2 = \max \{TT(2,1)-O_2,\ TT(2,1)-O_2+T_1+I_1-T_2-I_2$

$C_2 = O_2+I_2+T_2-TT(2,1)$

4. LAG-LAG/LAG-LEAD SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2)-I_2,\ TT(1,2)-I_2+T_1+O_1-T_2-O_2\}$

$B_1 = \max \{TT(1,2)-I_2,\ TT(1,2)-I_2+T_1+O_1-T_2-O_2\}$

$C_1 = I_2+O_2+T(2,4)-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1),\ TT(2,1)+T_1+I_1-T_2-I_2\}$

$B_2 = \max \{TT(2,1),\ TT(2,1)+T_1+I_1-T_2-I_2\}$

$C_2 = I_2+T_2-TT(2,1)$

5. LEAD-LEAD/LAG-LAG SEQUENCE COMBINATION

a. Outbound bandwidth

$A_1 = \min \{ TT(1,2)+L_1+I_1,\ TT(1,2)+L_1+I_1+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{ TT(1,2)+L_1+I_1,\ TT(1,2)+L_1+I_1+O_1+T_1-O_2-T_2\}$

$C_1 = T_2+O_2-L_1-I_1-TT(1,2)$

b. Inbound bandwidth

$A_2 = \min \{TT(2,1)+L_1+O_1,\ TT(2,1)+L_1+O_1+I_1+T_1-I_2-T_2 \}$

$B_2 = \max \{TT(2,1)+L_1+O_1,\ TT(2,1)+L_1+O_1+I_1+T_1-I_2-T_2 \}$

$C_2 = T(2,1)+I(2,1)-L(1,2)-O(1,2)-TT(2,1)$

6. LEAD-LEAD/LEAD-LEAD SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2)+L_1+I_1-L_2-I_2,\ TT(1,2)+L_1+I_1-L_2-I_2+O_1+T_1-O_2-T_2$

$B_1 = \max \{TT(1,2)+L_1+I_1-L_2-I_2,\ TT(1,2)+L_1+I_1-L_2-I_2+O_1+T_1-O_2-T_2$

$C_1\ L_2+O_2+I_2+T_2-L_1-I_1-TT(2,1)$

b. Inbound

$A_2 = \min\{TT(2,1)+L_1+O_1-L_2-O_2,\ TT(2,1)+L_1+O_1-L_2-O_2+I_1+T_1-I_2-T_2\}$

$B_2 = \max\{TT(2,1)+L_1+O_1-L_2-O_2,\ TT(2,1)+L_1+O_1-L_2-O_2+I_1+T_1-I_2-T_2\}$

$C_2 = L_2+O_2+I_2+T_2-L_1-O_1-TT(2,1)$

7. LEAD-LEAD/LEAD-LAG SEQUENCE COMBINATION

a. Outbound

$A_1 = \min\ \{TT(1,2)+L_1+I_1\ ,TT(1,2)+L_1+I_1+O_1+T_1-O_2-T_2\}$

$B_1 = \max\ \{TT(1,2)+L_1+I_1\ ,TT(1,2)+L_1+I_1+O_1+T_1-O_2-T_2\}$

$C_1 = O_2+T_2-L_1-I_1-TT(1,2)$

b. Inbound

$A_2 = \min\ \{TT(2,1)+L_1+O_1-O_2,\ TT(2,1)+L_1+O_1-O_2+I_1+T_1-I_2-T_2\}$

$B_2 = \max\ \{TT(2,1)+L_1+O_1-O_2,\ TT(2,1)+L_1+O_1-O_2+I_1+T_1-I_2-T_2\}$

$C_2 = O_2+I_2+T_2-L_1-O_1-TT(2,1)$

8. LEAD-LEAD/LAG-LEAD SEQUENCE COMBINATION

a. Outbound

$A_1 = \min\ \{TT(1,2)+L_1+I_1-I_2,TT(1,2)+L_1+I_1-I_2+O_1+T_1-O_2-T_2\}$

$B_1 = \max\ \{TT(1,2)+L_1+I_1-I_2,TT(1,2)+L_1+I_1-I_2+O_1+T_1-O_2-T_2\}$

$C_1 = I_2+O_2+T_2-L_1-I_1-TT(1,2)$

b. Inbound

$A_2 = \min\ \{TT(2,1)+L_1+O_1:,TT(2,1)+L_1+O_1+I_1+T_1-I_2-T_2\}$

$B_2 = \max\ \{TT(2,1)+L_1+O_1:,TT(2,1)+L_1+O_1+I_1+T_1-I_2-T_2\}$

$C_2 = I_2+T_2-L_1-O_1-TT(2,1)$

9. LEAD-LAG/LAG-LAG SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2), TT(1,2)+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2), TT(1,2)+O_1+T_1-O_2-T_2\}$

$C_1 = O_2+T_2-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1)+O_1, TT(2,1)+O_1+T_1+I_1-T_2-I_2\}$

$B_2 = \max \{TT(2,1)+O_1, TT(2,1)+O_1+T_1+I_1-T_2-I_2\}$

$C_2 = T_2+I_2-O_1-TT(2,1)$

10. LEAD-LAG/LEAD-LEAD SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2)-I_2, TT(1,2)-I_2+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2)-I_2, TT(1,2)-I_2+O_1+T_1-O_2-T_2\}$

$C_1 = I_2+O_2+T_2-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1)+O_1-O_2, TT(2,1)+O_1-O_2+T_1+I_1-T_2-I_2\}$

$B_2 = \max \{TT(2,1)+O_1-O_2, TT(2,1)+O_1-O_2+T_1+I_1-T_2-I_2\}$

$C_2 = O_2+T_2+I_2-O_1-TT(2,1)$

11. LEAD-LAG/LEAD-LAG SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2), TT(1,2)+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2), TT(1,2)+O_1+T_1-O_2-T_2\}$

$C_1 = O_2+T_2-TT(1,2)$

b. Inbound

A_2 = min $\{O_1-O_2+TT(2,1),\ T_1+I_1-I_2-T_2+O_1-O_2+TT(2,1)\}$

B_2 = max $\{O_1-O_2+TT(2,1),\ T_1+I_1-I_2-T_2+O_1-O_2+TT(2,1)\}$

$C_2 = O_2+I_2+T_2-O_1-TT(2,1)$

12. LEAD-LAG/LAG-LEAD SEQUENCE COMBINATION

a. Outbound

A_1 = min $\{TT(1,2)-I_2,\ TT(1,2)-I_2+O_1+T_1-T_2-O_2\}$

B_1 = max $\{TT(1,2)-I_2,\ TT(1,2)-I_2+O_1+T_1-T_2-O_2\}$

$C_1 = O_2+T_2+I_2-TT(1,2)$

b. Inbound

A_2 = min $\{TT(2,1)+O_1,\ TT(1,2)+O_1+T_1+I_1-T_2-I_2\}$

B_2 = max $\{TT(2,1)+O_1,\ TT(1,2)+O_1+T_1+I_1-T_2-I_2\}$

$C_2 = T_2+I_2-O_1-TT(2,1)$

13. LAG-LEAD/LAG-LAG SEQUENCE COMBINATION

a. Outbound

A_1 = min $\{TT(1,2)+I_1,TT(1,2)+I_1+O_1+T_1-O_2-T_2\}$

B_1= max $\{TT(1,2)+I_1,TT(1,2)+I_1+O_1+T_1-O_2-T_2\}$

$C_1 = T_2+O_2-I_1-TT(1,2)$

b. Inbound

A_2 = min $\{TT(2,1),\ TT(2,1)+T_1-I_2-T_2\}$

B_2 = max $\{TT(2,1),\ TT(2,1)+T_1-I_2-T_2\}$

$C_2 = T_2+I_2-TT(2,1)$

14. LAG-LEAD/LEAD-LEAD SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2)+I_1-L_2-I_2,\ TT(1,2)+I_1-L_2-I_2+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2)+I_1-L_2-I_2,\ TT(1,2)+I_1-L_2-I_2+O_1+T_1-O_2-T_2\}$

$C_1 = L_2+I_2+O_2+T_2-I_1-T_1-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1)-L_2-O_2,\ TT(2,1)-L_2-O_2+I_1+T_1-I_2-T_2\}$

$B_2 = \max \{TT(2,1)-L_2-O_2,\ TT(2,1)-L_2-O_2+I_1+T_1-I_2-T_2\}$

$C_2 = L_2+O_2+I_2+T_2-TT(2,1)$

15. LAG-LEAD/LEAD-LAG SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2)-I_1,\ TT(1,2)-I_1+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2)-I_1,\ TT(1,2)-I_1+O_1+T_1-O_2-T_2\}$

$C_1 = O_2+T_2-I_1-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1)-O_2,\ TT(2,1)-O_2+I_1+T_1-I_2-T_2\}$

$B_2 = \max \{TT(2,1)-O_2,\ TT(2,1)-O_2+I_1+T_1-I_2-T_2\}$

$C_2 = O_2+I_2+T_2-TT(2,1)$

16. LAG-LEAD/LAG-LEAD SEQUENCE COMBINATION

a. Outbound

$A_1 = \min \{TT(1,2)+I_1-I_2,\ TT(1,2)+I_1-I_2+O_1+T_1-O_2-T_2\}$

$B_1 = \max \{TT(1,2)+I_1-I_2,\ TT(1,2)+I_1-I_2+O_1+T_1-O_2-T_2\}$

$C_1 = I_2+O_2+T_2-I_1-TT(1,2)$

b. Inbound

$A_2 = \min \{TT(2,1),\ TT(2,1)+I_1+T_1-I_2-T_2\}$

$B_2 = \max \{TT(2,1),\ TT(2,1)+I_1+T_1-I_2-T_2\}$

$C_2 = I_2+T_2-TT(2,1)$

APPENDIX B

PROGRAM LISTING

```
110     INPUT "NUMBER OF SIGNALS ";N
120     INPUT "CYCLE LENGTH ";C
130     REM
        ***************************************************
140     REM * 1 = LAG-LAG, 2 = LEAD-LEAD, 3 = LEAD-LAG, AND
        4 = LAG-LEAD*
150     REM
        **************************************************
160     DIM OBL(N):DIM TT(N,N):DIM IBL(N):DIM OBT(N):DIM
        IBT(N):DIM CL(N)
170     DIM A(2,16):DIM B(2,16):DIM C(2,16):DIM
        X(2,N,4):DIM Y(2,N,4):DIM Z(2,N,4)
180     DIM OFFSET(N):DIM OPTPAT(N):DIM FL(N,4):DIM
        PS(16):DIM PZ(N,4)
190     DIM V(2,16):DIM U(2,16):DIM W(2,N,4):DIM
        Q(2,N,4):DIM FA(16)
200     DIM MAXB(16),OFFB(16),OFFE(16):DIM VARB(N,4):DIM
        VARE(N,4):DIM GRBD(N,4)
205     DIM BOB(N):DIM EOB(N):DIM BIB(N):DIM EIB(N)
210     DIM
        O(N,4),I(N,4),T(N,4),L(N,4),OB(N,4),OE(N,4),IB(N,4)
        ,IE(N,4)
220     ICURMIN=10000:OCURMIN=10000:ICURMAX=0:OCURMAX=0
230     FOR Q = 1 TO N
235     FOR J=1 TO N
240     TT(J,Q) = 0
245     NEXT J
250     NEXT Q
260     FOR I=1 TO N-1
270     PRINT "TRAVEL TIME SIGNAL";I;"TO SIGNAL";I+1;
280     INPUT TT(I,I+1)
290     PRINT "TRAVEL TIME SIGNAL";I+1;" TO SIGNAL";I;
300     INPUT TT(I+1,I)
310     NEXT I
315     FOR Z=1 TO N-2
320     FOR A= Z+2 TO N
330     FOR B = Z TO A-1
340     TT(Z,A)=TT(Z,A)+TT(B,B+1)
350     TT(A,Z)=TT(A,Z)+TT(B+1,B)
360     NEXT B
370     NEXT A
375     NEXT Z
```

```
380     FOR J=1 TO N
385     FOR Z = 1 TO N
390     IF TT(Z,J)>C THEN TT(Z,J)=TT(Z,J)-C:GOTO 390
400     IF TT(J,Z)>C THEN TT(J,Z)=TT(J,Z)-C:GOTO 400
405     NEXT Z
410     NEXT J
420     FOR I=1 TO N
430         PRINT "OUTBOUND LEFT,INBOUND LEFT,OUTBOUND
            THRU,INBOUND THRU FOR SIGNAL";I
440         INPUT OBL(I), IBL(I), OBT(I),IBT(I)
450         INPUT "NUMBER SECONDS GREEN ALLOCATED TO
            ARTERIAL AT THIS SIGNAL ";CL(I)
460     NEXT I
540     FOR I=1 TO N
550         O(I,3)=OBL(I)
560         I(I,3)=IBL(I)
570         T(I,3)=CL(I)-O(I,3)-I(I,3)
580         IF T(I,3)<0 THEN T(I,3)=CL(I):O(I,3)=0:I(I,3)=0
590         O(I,4)=O(I,3)
600         I(I,4)=I(I,3)
610         T(I,4)=T(I,3)
620         IF OBL(I)<IBL(I) THEN L(I,2)=OBL(I) ELSE
            L(I,2)=IBL(I)
630         IF IBL(I)-L(I,2)>0 THEN I(I,2)=(IBL(I)-L(I,2))
            ELSE I(I,2)=0
640         IF OBL(I)-L(I,2)>0 THEN O(I,2)=(OBL(I)-L(I,2))
            ELSE O(I,2)=0
650         T(I,2)=CL(I)-L(I,2)-I(I,2)-O(I,2)
660         IF T(I,2)<0 THEN
            T(I,2)=CL(I):O(I,2)=0:I(I,2)=0:L(I,2)=0
670         L(I,1)=L(I,2):I(I,1)=I(I,2):O(I,1)=O(I,2):T(I,1
            )=T(I,2)
680     NEXT I
690     REM LELE/LALA ROUTINE
700     IF O(1,2)+T(1,2)>O(N,1)+T(N,1) THEN
        A(1,5)=TT(1,N)+L(1,2)+I(1,2):B(1,5)=TT(1,N)+L(1,2)+
        I(1,2)+O(1,2)+T(1,2)-O(N,1)-T(N,1) ELSE
        A(1,5)=TT(1,N)+L(1,2)+I(1,2)+O(1,2)+T(1,2)-O(N,1)-T
        (N,1):B(1,5)=TT(1,N)+L(1,2)+I(1,2)
710     C(1,5)=T(N,1)+O(N,1)-L(1,2)-I(1,2)-TT(1,N)
720     IF I(1,2)+T(1,2)>I(N,1)+T(N,1) THEN
        A(2,5)=C-TT(N,1)+L(1,2)+O(1,2):B(2,5)=C-TT(N,1)+L(1
        ,2)+O(1,2)+I(1,2)+T(1,2)-I(N,1)-T(N,1) ELSE
        A(2,5)=C-TT(N,1)+L(1,2)+O(1,2)+I(1,2)+T(1,2)-I(N,1)
        -T(N,1):B(2,5)=C-TT(N,1)+L(1,2)+O(1,2)
730     C(2,5)=T(N,1)+I(N,1)-L(1,2)-O(1,2)+TT(N,1)-C
740     REM LELE/LELE ROUTINE
750     IF O(1,2)+T(1,2)>O(N,2)+T(N,2) THEN
        A(1,6)=TT(1,N)+L(1,2)+I(1,2)-L(N,2)-I(N,2):
        B(1,6)=A(1,6)+O(1,2)+T(1,2)-O(N,2)-T(N,2) ELSE
        B(1,6)=TT(1,N)+L(1,2)+I(1,2)-L(N,2)-I(N,2):A(1,6)=B
        (1,6)+O(1,2)+T(1,2)-O(N,2)-T(N,2)
```

```
760     C(1,6)=L(N,2)+O(N,2)+I(N,2)+T(N,2)-L(1,2)-I(1,2)-TT
        (1,N)
770     IF I(1,2)+T(1,2)>I(N,2)+T(N,2) THEN
        A(2,6)=C-TT(N,1)+L(1,2)+O(1,2)-L(N,2)-O(N,2):
        B(2,6)=A(2,6)+I(1,2)+T(1,2)-I(N,2)-T(N,2) ELSE
        B(2,6)=C-TT(N,1)+L(1,2)+O(1,2)-L(N,2)-O(N,2):A(2,6)
        =B(2,6)+I(1,2)+T(1,2)-I(N,2)-T(N,2)
780     C(2,6)=L(N,2)+O(N,2)+I(N,2)+T(N,2)-L(1,2)-O(1,2)+TT
        (N,1)-C
790     REM LELE/LELA ROUTINE
800     IF O(1,2)+T(1,2)>O(N,3)+T(N,3) THEN
        A(1,7)=TT(1,N)+L(1,2)+I(1,2):B(1,7)=A(1,7)+O(1,2)+
        T(1,2)-O(N,3)-T(N,3) ELSE
        B(1,7)=TT(1,N)+L(1,2)+I(1,2):A(1,7)=B(1,7)+O(1,2)+T
        (1,2)-O(N,3)-T(N,3)
810     C(1,7)=O(N,3)+T(N,3)-L(1,2)-I(1,2)-TT(1,N)
820     IF I(1,2)+T(1,2)>I(N,3)+T(N,3)THEN
        A(2,7)=C-TT(N,1)+L(1,2)+O(1,2)-O(N,3):B(2,7)=A(2,7)
        +I(1,2)+T(1,2)-I(N,3)-T(N,3) ELSE
        B(2,7)=C-TT(N,1)+L(1,2)+O(1,2)-O(N,3):A(2,7)=B(2,7)
        +I(1,2)+T(1,2)-I(N,3)-T(N,3)
830     C(2,7)=O(N,3)+I(N,3)+T(N,3)-L(1,2)-O(1,2)+TT(N,1)-C
840     REM LELE/LALE ROUTINE
850     IF O(1,2)+T(1,2)>O(N,4)+T(N,4) THEN
        A(1,8)=TT(1,N)+L(1,2)+I(1,2)-I(N,4):B(1,8)=A(1,8)+
        O(1,2)+T(1,2)-O(N,4)-T(N,4) ELSE
        B(1,8)=TT(1,N)+L(1,2)+I(1,2)-I(N,4):A(1,8)=B(1,8)+O
        (1,2)+T(1,2)-O(N,4)-T(N,4)
860     C(1,8)=I(N,4)+O(N,4)+T(N,4)-L(1,2)-I(1,2)-TT(1,N)
870     IF I(1,2)+T(1,2)>I(N,4)+T(N,4) THEN
        A(2,8)=C-TT(N,1)+L(1,2)+O(1,2):B(2,8)=A(2,8)+I(1,2)
        +T(1,2)-I(N,4)-T(N,4) ELSE
        B(2,8)=C-TT(N,1)+L(1,2)+O(1,2):A(2,8)=B(2,8)+I(1,2)
        +T(1,2)-I(N,4)-T(N,4)
880     C(2,8)=I(N,4)+T(N,4)-L(1,2)-O(1,2)+TT(N,1)-C
890     REM LALA/LALA ROUTINE
900     IF T(1,1)+O(1,1)>T(N,1)+O(N,1)
        A(1,1)=TT(1,N):B(1,1)=TT(1,N)+O(1,1)+T(1,1)-O(N,1)-
        T(N,1)  ELSE
        A(1,1)=TT(1,N)+O(1,1)+T(1,1)-O(N,1)-T(N,1):B(1,1)=T
        T(1,N)
910     C(1,1)=T(N,1)+O(N,1)-TT(1,N)
920     IF T(1,1)+I(1,1)>T(N,1)+I(N,1) THEN
        A(2,1)=C-TT(N,1):B(2,1)=C-TT(N,1)+T(1,1)+I(1,1)-T(N
        ,1)-I(N,1) ELSE
        A(2,1)=C-TT(N,1)+T(1,1)+I(1,1)-T(N,1)-I(N,1):B(2,1)
        =C-TT(N,1)
930     C(2,1)=T(N,1)+I(N,1)+TT(N,1)-C
940     REM LALA/LELE ROUTINE
950     IF T(1,1)+O(1,1)>T(N,2)+O(N,2) THEN
        A(1,2)=TT(1,N)-I(N,2)-L(N,2):B(1,2)=TT(1,N)-I(N,2)-
        L(N,2)+O(1,1)+T(1,1)-O(N,2)-T(N,2) ELSE
        A(1,2)=TT(1,N)-I(N,2)-L(N,2)+O(1,1)+T(1,1)-O(N,2)-T
        (N,2):B(1,2)=TT(1,N)-I(N,2)-L(N,2)
960     C(1,2)=I(N,2)+L(N,2)+O(N,2)+T(N,2)-TT(1,N)
970     IF T(1,1)+I(1,1)>T(N,2)+I(N,2) THEN
        A(2,2)=C-TT(N,1)-L(N,2)-O(N,2):B(2,2)=C-TT(N,1)-L(N
        ,2)-O(N,2)+T(1,1)+I(1,1)-T(N,2)-I(N,2) ELSE
        A(2,2)=C-TT(N,1)-L(N,2)-O(N,2)+T(1,1)+I(1,1)-T(N,2)
        -I(N,2):B(2,2)=C-TT(N,1)-L(N,2)-O(N,2)
980     C(2,2)=I(N,2)+L(N,2)+O(N,2)+T(N,2)+TT(N,1)-C
990     REM LALA/LELA ROUTINE
1000    IF T(1,1)+O(1,1)>T(N,3)+O(N,3) THEN
        A(1,3)=TT(1,N):B(1,3)=TT(1,N)+O(1,1)+T(1,1)-O(N,3)-
        T(N,3) ELSE
        A(1,3)=TT(1,N)+T(1,1)+O(1,1)-T(N,3)-O(N,3):B(1,3)=T
        T(1,N)
1010    C(1,3)=O(N,3)+T(N,3)-TT(1,N)
1020    IF T(1,1)+I(1,1)>T(N,3)+I(N,3) THEN
        A(2,3)=C-TT(N,1)-O(N,3):B(2,3)=C-TT(N,1)-O(N,3)+T(1
        ,1)+I(1,1)-T(N,3)-I(N,3) ELSE
        A(2,3)=C-TT(N,1)-O(N,3)+T(1,1)+I(1,1)-T(N,3)-I(N,3)
        :B(2,3)=C-TT(N,1)-O(N,3)
1030    C(2,3)=O(N,3)+I(N,3)+T(N,3)+TT(N,1)-C
1040    REM LALA/LALE ROUTINE
1050    IF T(1,1)+O(1,1)>T(N,4)+O(N,4) THEN
        A(1,4)=TT(1,N)-I(N,4):B(1,4)=TT(1,N)-I(N,4)+T(1,1)+
        O(1,1)-T(N,4)-O(N,4) ELSE
        A(1,4)=TT(1,N)-I(N,4)+T(1,1)+O(1,1)-T(N,4)-O(N,4):B
        (1,4)=TT(1,N)-I(N,4)
1060    C(1,4)=I(N,4)+O(N,4)+T(N,4)-TT(1,N)
1070    IF T(1,1)+I(1,1)>T(N,4)+I(N,4) THEN
        A(2,4)=C-TT(N,1):B(2,4)=C-TT(N,1)+T(1,1)+I(1,1)-T(N
        ,4)-I(N,4) ELSE A(2,4)=
        C-TT(N,1)+T(1,1)+I(1,1)-T(N,4)-I(N,4):B(2,4)=C-TT(N
        ,1)
1080    C(2,4)=I(N,4)+T(N,4)+TT(N,1)-C
1090    REM LELA/LALA ROUTINE
1100    IF O(1,3)+T(1,3)>O(N,1)+T(N,1) THEN
        A(1,9)=TT(1,N):B(1,9)=TT(1,N)+O(1,3)+T(1,3)-O(N,1)-
        T(N,1) ELSE
        A(1,9)=TT(1,N)+O(1,3)+T(1,3)-O(N,1)-T(N,1):B(1,9)=T
        T(1,N)
1110    C(1,9)=O(N,1)+T(N,1)-TT(1,N)
1120    IF T(1,3)+I(1,3)>T(N,1)+I(N,1) THEN
        A(2,9)=C-TT(N,1)+O(1,3):B(2,9)=C-TT(N,1)+O(1,3)+T(1
        ,3)+I(1,3)-T(N,1)-I(N,1) ELSE
        A(2,9)=C-TT(N,1)+O(1,3)+T(1,3)+I(1,3)-T(N,1)-I(N,1)
        :B(2,9)=C-TT(N,1)+O(1,3)
1130    C(2,9)=T(N,1)+I(N,1)-O(1,3)+TT(N,1)-C
1140    REM  LELA/LELE ROUTINE
1150    IF O(1,3)+T(1,3)>O(N,2)+T(N,2) THEN
        A(1,10)=TT(1,N)-I(N,2):B(1,10)=TT(1,N)-I(N,2)+O(1,3
        )+T(1,3)-O(N,2)-T(N,2) ELSE
        A(1,10)=TT(1,N)-I(N,2)+O(1,3)+T(1,3)-O(N,2)-T(N,2):
        B(1,10)=TT(1,N)-I(N,2)
1160    C(1,10)=I(N,2)+O(N,2)+T(N,2)-TT(1,N)
1170    IF T(1,3)+I(1,3) > T(N,2)+I(N,2) THEN
        A(2,10)=C-TT(N,1)+O(1,3)-O(N,2):B(2,10)=C-TT(N,1)+O
        (1,3)-O(N,2)+T(1,3)+I(1,3)-T(N,2)-I(N,2) ELSE
        A(2,10)=C-TT(N,1)+O(1,3)-O(N,2)+T(1,3)+I(1,3)-T(N,2
        )-I(N,2):B(2,10)=C-TT(N,1)+O(1,3)-O(N,2)
1180    C(2,10)=O(N,2)+T(N,2)+I(N,2)-O(1,3)+TT(N,1)-C
1190    REM LELA/LALE ROUTINE
1200    IF O(1,3)+T(1,3)>O(N,4)+T(N,4) THEN
        A(1,12)=TT(1,N)-I(N,4):B(1,12)=TT(1,N)-I(N,4)+O(1,3
        )+T(1,3)-T(N,4)-O(N,4) ELSE
        A(1,12)=TT(1,N)-I(N,4)+O(1,3)+T(1,3)-O(N,4)-T(N,4):
        B(1,12)=TT(1,N)-I(N,4)
1210    C(1,12)=O(N,4)+T(N,4)+I(N,4)-TT(1,N)
1220    IF T(1,3)+I(1,3)>T(N,4)+I(N,4) THEN
        A(2,12)=C-TT(N,1)+O(1,3):B(2,12)=C-TT(N,1)+O(1,3)+T
        (1,3)+I(1,3)-T(N,4)-I(N,4) ELSE
        A(2,12)=C-TT(N,1)+O(1,3)+T(1,3)+I(1,3)-T(N,4)-I(N,4
        ):B(2,12)=C-TT(N,1)+O(1,3)
1230    C(2,12)=T(N,4)+I(N,4)-O(1,3)+TT(N,1)-C
1240    REM LELA/LELA ROUTINE
1250    IF (O(1,3) +T(1,3) > O(N,3)+T(N,3)) THEN
        A(1,11) =
        TT(1,N):B(1,11)=TT(1,N)+O(1,3)+T(1,3)-O(N,3)-T(N,3)
        ELSE
        A(1,11)=TT(1,N)+O(1,3)+T(1,3)-O(N,3)-T(N,3):B(1,11)
        =TT(1,N)
1260    C(1,11)=O(N,3)+T(N,3)-TT(1,N)
1270    IF I(1,3)+T(1,3) > I(N,3)+T(N,3) THEN A(2,11) =
        O(1,3)-O(N,3)+C-TT(N,1):B(2,11)=T(1,3)+I(1,3)-I(N,3
        )-T(N,3)+O(1,3)-O(N,3)+C-TT(N,1) ELSE
        A(2,11)=T(1,3)+I(1,3)-T(N,3)-I(N,3)+O(1,3)-O(N,3)+C
        -TT(N,1):B(2,11)=O(1,3)-O(N,3)+C-TT(N,1)
1280    C(2,11)=O(N,3)+I(N,3)+T(N,3)-O(1,3)+TT(N,1)-C
1290    REM LALE/LALA ROUTINE
1300    IF O(1,4)+T(1,4)>O(N,1)+T(N,1) THEN
        A(1,13)=TT(1,N)+I(1,4):B(1,13)=A(1,13)+O(1,4)+T(1,4
        )-O(N,1)-T(N,1) ELSE
        B(1,13)=TT(1,N)+I(1,4):A(1,13)=B(1,13)+O(1,4)+T(1,4
        )-O(N,1)-T(N,1)
1310    C(1,13)=T(N,1)+O(N,1)-I(1,4)-TT(1,N)
1320    IF I(1,4)+T(1,4)>I(N,1)+T(N,1) THEN
        A(2,13)=C-TT(N,1):B(2,13)=A(2,13)+I(1,4)+T(1,4)-I(N
        ,1)-T(N,1) ELSE
        B(2,13)=C-TT(N,1):A(2,13)=B(2,13)+I(1,4)+T(1,4)-I(N
        ,1)-T(N,1)
1330    C(2,13)=T(N,1)+I(N,1)+TT(N,1)-C
1340    REM LALE/LELE ROUTINE
1350    IF O(1,4)+T(1,4)>O(N,2)+T(N,2) THEN
        A(1,14)=TT(1,N)+I(1,4)-L(N,2)-I(N,2):B(1,14)=A(1,14
        )+O(1,4)+T(1,4)-O(N,2)-T(N,2) ELSE
        B(1,14)=TT(1,N)+I(1,4)-L(N,2)-I(N,2):A(1,14)=B(1,14
        )+O(1,4)+T(1,4)-O(N,2)-T(N,2)
1360    C(1,14)=L(N,2)+I(N,2)+O(N,2)+T(N,2)-I(1,4)-T(1,4)-T
        T(1,N)
1370    IF I(1,4)+T(1,4)>I(N,2)+T(N,2) THEN
        A(2,14)=C-TT(N,1)-L(N,2)-O(N,2):B(2,14)=A(2,14)+I(1
        ,4)+T(1,4)-I(N,2)-T(N,2) ELSE
        B(2,14)=C-TT(N,1)-L(N,2)-O(N,2):A(2,14)=B(2,14)+I(1
        ,4)+T(1,4)-I(N,2)-T(N,2)
1380    C(2,14)=L(N,2)+O(N,2)+I(N,2)+T(N,2)+TT(N,1)-C
1390    REM LALE/LELA ROUTINE
1400    IF O(1,4)+T(1,4)>O(N,3)+T(N,3) THEN
        A(1,15)=TT(1,N)-I(1,4):B(1,15)=A(1,15)+O(1,4)+T(1,4
        )-O(N,3)-T(N,3) ELSE
        B(1,15)=TT(1,N)-I(1,4):A(1,15)=B(1,15)+O(1,4)+T(1,4
        )-O(N,3)-T(N,3)
1410    C(1,15)=O(N,3)+T(N,3)-I(1,4)-TT(1,N)
1420    IF I(1,4)+T(1,4)>I(N,3)+T(N,3) THEN
        A(2,15)=C-TT(N,1)-O(N,3):B(2,15)=A(2,15)+I(1,4)+T(1
        ,4)-I(N,3)-T(N,3) ELSE
        B(2,15)=C-TT(N,1)-O(N,3):A(2,15)=B(2,15)+I(1,4)+T(1
        ,4)-I(N,3)-T(N,3)
1430    C(2,15)=O(N,3)+I(N,3)+T(N,3)+TT(N,1)-C
1440    REM LALE/LALE ROUTINE
1450    IF O(1,4)+T(1,4)>O(N,4)+T(N,4) THEN
        A(1,16)=TT(1,N)+I(1,4)-I(N,4):B(1,16)=A(1,16)+O(1,4
        )+T(1,4)-O(N,4)-T(N,4) ELSE
        B(1,16)=TT(1,N)+I(1,4)-I(N,4):A(1,16)=B(1,16)+O(1,4
        )+T(1,4)-O(N,4)-T(N,4)
1460    C(1,16)=I(N,4)+O(N,4)+T(N,4)-I(1,4)-TT(1,N)
1470    IF I(1,4)+T(1,4)>I(N,4)+T(N,4) THEN
        A(2,16)=C-TT(N,1):B(2,16)=C-TT(N,1)+I(1,4)+T(1,4)-I
        (N,4)-T(N,4) ELSE
        B(2,16)=C-TT(N,1):A(2,16)=C-TT(N,1)+I(1,4)+T(1,4)-I
        (N,4)-T(N,4)
1480    C(2,16)=I(N,4)+T(N,4)+TT(N,1)-C
1540    LPRINT "MAXBAND AND OFFSET RESULTS"
1550    LPRINT TAB(28);"OFFSET RANGES FROM"
1560    LPRINT TAB(5);"MAX SUM OF BANDS";TAB(25);"OFFSET LB
        ";TAB(37);"TO";TAB(40)"OFFSET UB"
1570    FOR H=1 TO 16
1580    GOSUB 3830
1590    LPRINT
        TAB(5);MAXB(H);TAB(25);OFFB(H);TAB(40);OFFE(H)
1600    NEXT H
1660    GOSUB 1730
1670    FOR M=2 TO N-1
1680        GOSUB 3300
1690        GOSUB 2510
1700        GOSUB 3630
1705        GOSUB 3530
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1710    NEXT M
1720    END
1730    OPTBAND=-1
1740    FOR E = 1 TO 16
1750        IF OPTBAND> MAXB(E) THEN GOTO 1790
1760        OPTBAND=MAXB(E)
1770        OPTLB = OFFB(E)
1780        OPTUP = OFFE(E)
1790    NEXT E
1800    FOR E = 1 TO 16
1810        IF OPTBAND=MAXB(E) THEN GOSUB 1840
1820    NEXT E
1830    RETURN
1840    LPRINT "OFFSET RANGE FROM
        ";OFFB(E);"TO";OFFE(E);"YIELDS MAX SUM OF
        BANDS";MAXB(E)
1850    IF E <=4 THEN GOSUB 1980:GOTO 1890
1860    IF E <=8 THEN GOSUB 2040:GOTO 1890
1870    IF E <=12 THEN GOSUB 2100:GOTO 1890
1880    GOSUB 2160
1890    IF E MOD 4 = 1 THEN GOSUB 2220:GOTO 1930
1900    IF E MOD 4 = 2 THEN GOSUB 2290:GOTO 1930
1910    IF E MOD 4 = 3 THEN GOSUB 2360:GOTO 1930
1920    IF E MOD 4 = 0 THEN GOSUB 2430
1930    IF SNORMAX>S1ORMAX THEN OCURMAX=SNORMAX ELSE
        OCURMAX=S1ORMAX
1940    IF SNIRMAX>S1IRMAX THEN ICURMAX=SNIRMAX ELSE
        ICURMAX=S1IRMAX
1950    IF SNORMIN<S1ORMIN THEN OCURMIN=SNORMIN ELSE
        OCURMIN=S1ORMIN
1960    IF SNIRMIN<S1IRMIN THEN ICURMIN=SNIRMIN ELSE
        ICURMIN=S1IRMIN
1970    RETURN
1980    LPRINT " LAG-LAG AT SIGNAL 1";
1990    OFFSET(1)=0
2000    OPTPAT(1) = 1:J=1:GOSUB 3240
2010    S1ORMAX =OFFSET(1):S1ORMIN=OFFSET(1)+T(1,1)+O(1,1)
2020    S1IRMAX=OFFSET(1):S1IRMIN=OFFSET(1)+T(1,1)+I(1,1)
2030    RETURN
2040    LPRINT " LEAD-LEAD AT SIGNAL 1";
2050    OFFSET(1)=0
2060    OPTPAT(1) = 2:J=1:GOSUB 3240
2070    S1ORMAX
        =OFFSET(1)+L(1,2)+I(1,2):S1ORMIN=OFFSET(1)+L(1,2)+I
        (1,2)+T(1,2)+O(1,2)
2080    S1IRMAX=OFFSET(1)+L(1,2)+O(1,2):S1IRMIN=OFFSET(1)+L
        (1,2)+O(1,2)+T(1,2)+I(1,2)
2090    RETURN
2100    LPRINT " LEAD-LAG AT SIGNAL 1";
2110    OFFSET(1)=0
2120    OPTPAT(1) = 3:J=1:GOSUB 3240
2130    S1ORMAX =OFFSET(1):S1ORMIN=OFFSET(1)+T(1,3)+O(1,3)
```

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2140    S1IRMAX=OFFSET(1)+O(1,3):S1IRMIN=OFFSET(1)+O(1,3)+T
        (1,3)+I(1,3)
2150    RETURN
2160    LPRINT " LAG-LEAD AT SIGNAL 1";
2170    OFFSET(1)=0
2180    OPTPAT(1) = 4:J=1:GOSUB 3240
2190    S1ORMAX
        =OFFSET(1)+I(1,4):S1ORMIN=OFFSET(1)+I(1,4)+T(1,4)+O
        (1,4)
2200    S1IRMAX=OFFSET(1):S1IRMIN=OFFSET(1)+T(1,4)+I(1,4)
2210    RETURN
2220    LPRINT " LAG-LAG AT SIGNAL";N
2230    LPRINT
2240    OFFSET(N)=OPTLB
2250    OPTPAT(N) = 1:J=N:GOSUB 3240
2260    SNORMAX
        =OFFSET(N)-TT(1,N):SNORMIN=OFFSET(N)+T(N,1)+O(N,1)-
        TT(1,N)
2270    SNIRMAX=OFFSET(N)+TT(N,1)-C:SNIRMIN=OFFSET(N)+T(N,1
        )+I(N,1)+TT(N,1)-C
2280    RETURN
2290    LPRINT " LEAD-LEAD AT SIGNAL";N
2300    LPRINT
2310    OFFSET(N)=OPTLB
2320    OPTPAT(N) = 2:J=N:GOSUB 3240
2330    SNORMAX=OFFSET(N)+L(N,2)+I(N,2)-TT(1,N):
        SNORMIN=OFFSET(N)+L(N,2)+I(N,2)+T(N,2)+O(N,2)-TT(1,
        N)
2340    SNIRMAX=OFFSET(N)+L(N,2)+O(N,2)+TT(N,1)-C:
        SNIRMIN=OFFSET(N)+L(N,2)+O(N,2)+T(N,2)+I(N,2)+TT(N,
        1)-C
2350    RETURN
2360    LPRINT " LEAD-LAG AT SIGNAL";N
2370    LPRINT
2380    OFFSET(N)=OPTLB
2390    OPTPAT(N) = 3:J=N:GOSUB 3240
2400    SNORMAX
        =OFFSET(N)-TT(1,N):SNORMIN=OFFSET(N)+T(N,3)+O(N,3)-
        TT(1,N)
2410    SNIRMAX=OFFSET(N)+O(N,3)+TT(N,1)-C:SNIRMIN=OFFSET(N
        )+O(N,3)+T(N,3)+I(N,3)+TT(N,1)-C
2420    RETURN
2430    LPRINT " LAG-LEAD AT SIGNAL";N
2440    LPRINT
2450    OFFSET(N)=OPTLB
2460    OPTPAT(N) = 4:J=N:GOSUB 3240
2470    SNORMAX=OFFSET(N)+I(N,4)-TT(1,N):SNORMIN=OFFSET(N)+
        I(N,4)+T(N,4)+O(N,4)-TT(1,N)
2480    SNIRMAX=OFFSET(N)+TT(N,1)-C:SNIRMIN=OFFSET(N)+T(N,4
        )+I(N,4)+TT(N,1)-C
2490    RETURN
2500    END
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2510      REM CALCULATE MAXIMUM GREENBAND AND OFFSETS
2520      R=M
2530      FOR T = 1 TO 4
2540          FL(R,T)=0: PZ(R,T)=0:REM CHECK FOR 0 BANDWIDTH
2550          IF X(1,R,T)<0 THEN
          Z(1,R,T)=X(1,R,T)+Z(1,R,T):X(1,R,T)=0
2560          IF X(2,R,T)<0 THEN
          Z(2,R,T)=X(2,R,T)+Z(2,R,T):X(2,R,T)=0
2570          IF X(1,R,T)+Z(1,R,T)<0 THEN
          X(1,R,T)=0:Z(1,R,T)=0:Y(1,R,T)=C
2580          IF Y(2,R,T)<0 THEN
          Z(2,R,T)=Y(2,R,T)+Z(2,R,T):Y(2,R,T)=0
2590          IF X(2,R,T)+Z(2,R,T)<0 THEN
          X(2,R,T)=0:Z(2,R,T)=0:Y(2,R,T)=C
2600          IF Y(1,R,T)<0 THEN
          Z(1,R,T)=Y(1,R,T)+Z(1,R,T):Y(1,R,T)=0
2610          IF Z(1,R,T)<0 THEN W(1,R,T)=-Z(1,R,T) ELSE
          W(1,R,T)=0
2620          IF X(1,R,T)+Y(1,R,T)+Z(1,R,T) < C THEN
          Q(1,R,T)=X(1,R,T)+Y(1,R,T)+Z(1,R,T) ELSE
              Q(1,R,T)=C
2630          IF Z(2,R,T)<0 THEN W(2,R,T)=-Z(2,R,T) ELSE
          W(2,R,T)=0
2640          IF X(2,R,T)+Y(2,R,T)+Z(2,R,T) < C THEN
          Q(2,R,T)=X(2,R,T)+Y(2,R,T)+Z(2,R,T) ELSE
              Q(2,R,T)=C
2650          IF W(2,R,T)<= W(1,R,T) THEN
          PZ(R,T)=PZ(R,T)+1000 ELSE IF W(2,R,T)<X(1,R,T)
          THEN PZ(R,T)=PZ(R,T)+2000 ELSE IF
          W(2,R,T)<Y(1,R,T) THEN PZ(R,T)=PZ(R,T)+3000   ELSE
          IF W(2,R,T)<Q(1,R,T) THEN    PZ(R,T)=PZ(R,T)+4000
          ELSE PZ(R,T)=PZ(R,T)+5000
2660      IF X(2,R,T)< W(1,R,T) THEN PZ(R,T)=PZ(R,T)+100 ELSE
          IF X(2,R,T)<X(1,R,T) THEN PZ(R,T)=PZ(R,T)+200 ELSE
          IF X(2,R,T)<Y(1,R,T) THEN PZ(R,T)=PZ(R,T)+300 ELSE
          IF X(2,R,T)<Q(1,R,T) THEN PZ(R,T)=PZ(R,T)+400 ELSE
          PZ(R,T)=PZ(R,T)+500
2670      IF Y(2,R,T)< W(1,R,T) THEN PZ(R,T)=PZ(R,T)+10
          ELSE IF Y(2,R,T)<X(1,R,T) THEN PZ(R,T)=PZ(R,T)+20
          ELSE IF Y(2,R,T)<Y(1,R,T) THEN PZ(R,T)=PZ(R,T)+30
          ELSE IF Y(2,R,T)<Q(1,R,T) THEN PZ(R,T)=PZ(R,T)+40
          ELSE PZ(R,T)=PZ(R,T)+50
2680      IF Q(2,R,T)< W(1,R,T) THEN PZ(R,T)=PZ(R,T)+1
          ELSE IF Q(2,R,T)<X(1,R,T) THEN PZ(R,T)=PZ(R,T)+2
          ELSE IF Q(2,R,T)<Y(1,R,T) THEN PZ(R,T)=PZ(R,T)+3
          ELSE IF Q(2,R,T)<Q(1,R,T) THEN PZ(R,T)=PZ(R,T)+4
          ELSE PZ(R,T)=PZ(R,T)+5
2690      IF PZ(R,T)=1111 THEN IF
          X(1,R,T)+Z(1,R,T)>X(2,R,T)+Z(2,R,T) THEN
          GRBD(R,T)=X(1,R,T)+Z(1,R,T):VARB(R,T)=X(1,R,T):VARE
          (R,T)=Y(1,R,T) ELSE
```

```
        GRBD(R,T)=X(2,R,T)+Z(2,R,T):VARB(R,T)=X(2,R,T):VARE
        (R,T)=Y(2,R,T)
2700    IF PZ(R,T)=1112 THEN GRBD(R,T)=
        X(1,R,T)+Z(1,R,T):VARB(R,T)=X(1,R,T):
        VARE(R,T)=Y(1,R,T)
2710    IF PZ(R,T)=1113 THEN GRBD(R,T)=
        X(2,R,T)+Z(2,R,T):VARB(R,T)=X(2,R,T):
        VARE(R,T)=Y(2,R,T)
2720    IF PZ(R,T)=1114 THEN GRBD(R,T)=
        X(2,R,T)+Z(2,R,T):VARB(R,T)=X(2,R,T):
        VARE(R,T)=Y(2,R,T)
2730    IF PZ(R,T)=1115 THEN GRBD(R,T)=
        X(2,R,T)+Z(2,R,T):VARB(R,T)=X(2,R,T):
        VARE(R,T)=Y(2,R,T)
2740    IF PZ(R,T)=1122 THEN GRBD(R,T)=
        X(1,R,T)+Z(1,R,T):VARB(R,T)=X(1,R,T):VARE(R,T)=Y(1,
        R,T)
2750    IF PZ(R,T)=1123 THEN GRBD(R,T)=
        X(2,R,T)+Y(2,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=Y(2,R
        ,T):VARE(R,T)=X(1,R,T)
2760    IF PZ(R,T)=1124 THEN GRBD(R,T)=
        X(2,R,T)+Y(2,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=Y(2,R
        ,T):VARE(R,T)=X(1,R,T)
2770    IF PZ(R,T)=1125 THEN GRBD(R,T)=
        X(2,R,T)+Y(2,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=Y(2,R
        ,T):VARE(R,T)=X(1,R,T)
2780    IF PZ(R,T)=1133 THEN GRBD(R,T)=
        X(2,R,T)+X(1,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=X(1,R
        ,T):VARE(R,T)=Y(2,R,T)
2790    IF PZ(R,T)=1134 THEN GRBD(R,T)=
        X(2,R,T)+X(1,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=X(1,R
        ,T):VARE(R,T)=Y(2,R,T)
2800    IF PZ(R,T)=1135 THEN GRBD(R,T)=
        X(2,R,T)+X(1,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=X(1,R
        ,T):VARE(R,T)=Y(1,R,T)
2810    IF PZ(R,T)=1144 THEN GRBD(R,T)=
        X(2,R,T)+X(1,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=X(1,R
        ,T):VARE(R,T)=Y(1,R,T)
2820    IF PZ(R,T)=1145 THEN GRBD(R,T)=
        X(2,R,T)+X(1,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=X(1,R
        ,T):VARE(R,T)=Y(1,R,T)
2830    IF PZ(R,T)=1155 THEN GRBD(R,T)=
        X(2,R,T)+X(1,R,T)+Z(2,R,T)+Z(1,R,T):VARB(R,T)=X(1,R
        ,T):VARE(R,T)=Y(1,R,T)
2840    IF PZ(R,T)=1222 THEN
        GRBD(R,T)=X(1,R,T)+Z(1,R,T):VARB(R,T)=X(1,R,T):VARE
        (R,T)=Y(1,R,T)
2850    IF PZ(R,T)=1223 THEN
        GRBD(R,T)=X(2,R,T)+Y(2,R,T)+Z(2,R,T)+Z(1,R,T):VARB(
        R,T)=Y(2,R,T):VARE(R,T)=X(1,R,T)
```

```
2860     IF PZ(R,T)=1224 THEN
         GRBD(R,T)=X(2,R,T)+Y(2,R,T)+Z(2,R,T)+Z(1,R,T):VARB(
         R,T)=Y(2,R,T):VARE(R,T)=X(1,R,T)
2870     IF PZ(R,T)=1225 THEN
         GRBD(R,T)=X(2,R,T)+Y(2,R,T)+Z(2,R,T)+Z(1,R,T):VARB(
         R,T)=Y(2,R,T):VARE(R,T)=X(1,R,T)
2880     IF PZ(R,T)=1233 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(1,R,T):VARE(R,T)=Y(2,R,T)
2890     IF PZ(R,T)=1235 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(1,R,T):VARE(R,T)=Y(2,R,T)
2900     IF PZ(R,T)=1244 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(1,R,T):VARE(R,T)=Y(1,R,T)
2910     IF PZ(R,T)=1245 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(1,R,T):VARE(R,T)=Y(1,R,T)
2920     IF PZ(R,T)=1255 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(1,R,T):VARE(R,T)=Y(1,R,T)
2930     IF PZ(R,T)=1333 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(2,R,T):VARE(R,T)=Y(2,R,T)
2940     IF PZ(R,T)=1334 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(2,R,T):VARE(R,T)=Y(2,R,T)
2950     IF PZ(R,T)=1335 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(2,R,T):VARE(R,T)=Y(2,R,T)
2960     IF PZ(R,T)=1344 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(2,R,T):VARE(R,T)=Y(1,R,T)
2970     IF PZ(R,T)=1345 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(2,R,T):VARE(R,T)=Y(1,R,T)
2980     IF PZ(R,T)=1355 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T)+X(1,R,T)+Z(1,R,T):VARB(
         R,T)=X(2,R,T):VARE(R,T)=Y(1,R,T)
2990     IF PZ(R,T)=1444 THEN
         GRBD(R,T)=X(1,R,T)+Y(1,R,T)+Z(1,R,T)+Z(2,R,T):VARB(
         R,T)=Y(1,R,T):VARE(R,T)=X(2,R,T)
3000     IF PZ(R,T)=1445 THEN
         GRBD(R,T)=X(1,R,T)+Y(1,R,T)+Z(1,R,T)+Z(2,R,T):VARB(
         R,T)=Y(1,R,T):VARE(R,T)=X(2,R,T)
3010     IF PZ(R,T)=1455 THEN
         GRBD(R,T)=X(1,R,T)+Y(1,R,T)+Z(1,R,T)+Z(2,R,T):VARB(
         R,T)=Y(1,R,T):VARE(R,T)=X(2,R,T)
3020     IF PZ(R,T)=1555 THEN
         GRBD(R,T)=X(2,R,T)+Z(2,R,T):VARB(R,T)=X(2,R,T):VARE
         (R,T)=Y(2,R,T)
```

```
3030    IF PZ(R,T)>1555 THEN FL(R,T)=9:GOSUB 4360 ELSE GOTO
        3050
3040    IF FL(R,T)=9 THEN GOTO 2510
3050    NEXT T
3060    REM  RETURN
3070    RETURN
3080    HV1H=V(1,H)
3090    HA1H=A(1,H)
3100    HB1H=B(1,H)
3110    HU1H=U(1,H)
3120    HC1H=C(1,H)
3130    V(1,H)=V(2,H)
3140    A(1,H)=A(2,H)
3150    B(1,H)=B(2,H)
3160    U(1,H)=U(2,H)
3170    C(1,H)=C(2,H)
3180    V(2,H)=HV1H
3190    A(2,H)=HA1H
3200    B(2,H)=HB1H
3210    U(2,H)=HU1H
3220    C(2,H)=HC1H
3230    RETURN
3240    REM COMPUTE BEG AND END OF BANDS
3250    IF OPTPAT(J)=1 THEN
        BOB(J)=OFFSET(J):EOB(J)=OFFSET(J)+T(J,1)+O(J,1):BIB
        (J)=OFFSET(J):EIB(J)=OFFSET(J)+T(J,1)+I(J,1)
3260    IF OPTPAT(J)=2 THEN
        BOB(J)=OFFSET(J)+L(J,2)+I(J,2):EOB(J)=OFFSET(J)+L(J
        ,2)+I(J,2)+O(J,2)+T(J,2):BIB(J)=OFFSET(J)+L(J,2)+O(
        J,2):EIB(J)=OFFSET(J)+L(J,2)+I(J,2)+O(J,2)+T(J,2)
3270    IF OPTPAT(J)=3 THEN
        BOB(J)=OFFSET(J):EOB(J)=OFFSET(J)+O(J,3)+T(J,3):BIB
        (J)=OFFSET(J)+O(J,3):EIB(J)=OFFSET(J)+I(J,3)+O(J,3)
        +T(J,3)
3280    IF OPTPAT(J)=4 THEN
        BOB(J)=OFFSET(J)+I(J,4):EOB(J)=OFFSET(J)+I(J,4)+O(J
        ,4)+T(J,4):BIB(J)=OFFSET(J):EIB(J)=OFFSET(J)+I(J,4)
        +T(J,4)
3290    RETURN
```

```
3300    REM LAG-LAG ON MIDDLE SIGNAL
3310    IF T(M,1)+O(M,1)< OCURMIN-OCURMAX THEN
        X(1,M,1)=TT(1,M)+OCURMAX:Y(1,M,1)=TT(1,M)+OCURMIN-T
        (M,1)-O(M,1) ELSE
        X(1,M,1)=TT(1,M)+OCURMIN-T(M,1)-O(M,1):Y(1,M,1)=TT(
        1,M)+OCURMAX
3320    Z(1,M,1)=T(M,1)+O(M,1)-TT(1,M)-OCURMAX
3330    IF I(M,1)+T(M,1)<ICURMIN-ICURMAX THEN
        X(2,M,1)=C-TT(N,1)+TT(N,M)+ICURMAX:Y(2,M,1)=C-TT(N,
        1)+TT(N,M)+ICURMIN-I(M,1)-T(M,1) ELSE
        X(2,M,1)=C-TT(N,1)+TT(N,M)+ICURMIN-I(M,1)-T(M,1):Y(
        2,M,1)=C-TT(N,1)+TT(N,M)+ICURMAX
3340    Z(2,M,1)=T(M,1)+I(M,1)-TT(N,M)+TT(N,1)-C-ICURMAX
3350    REM LEAD-LEAD ON MIDDLE SIGNAL
3360    IF T(M,2)+O(M,2)< OCURMIN-OCURMAX THEN
        X(1,M,2)=TT(1,M)+OCURMAX-L(M,2)-I(M,2):Y(1,M,2)=TT(
        1,M)+OCURMIN-L(M,2)-I(M,2)-T(M,2)-O(M,2) ELSE
        X(1,M,2)=TT(1,M)+OCURMIN-L(M,2)-I(M,2)-O(M,2)-T(M,2
        ):Y(1,M,2)=TT(1,M)+OCURMAX-L(M,2)-I(M,2)
3370    Z(1,M,2)=L(M,2)+I(M,2)+T(M,1)+O(M,1)-TT(1,M)-OCURMA
        X
3380    IF I(M,2)+T(M,2)<ICURMIN-ICURMAX THEN
        X(2,M,2)=C-TT(N,1)+TT(N,M)+ICURMAX-L(M,2)-O(M,2):Y(
        2,M,2)=C-TT(N,1)+TT(N,M)+ICURMIN-L(M,2)-O(M,2)-I(M,
        2)-T(M,2) ELSE
        X(2,M,2)=C-TT(N,1)+TT(N,M)+ICURMIN-L(M,2)-O(M,2)-I(
        M,2)-T(M,2):Y(2,M,2)=C-TT(N,1)+TT(N,M)+ICURMAX-
        L(M,2)-O(M,2)
3390    Z(2,M,2)=L(M,2)+O(M,2)+I(M,2)+T(M,2)-TT(N,M)+TT(N,1
        )-C-ICURMAX
3400    REM LEAD-LAG ON MIDDLE SIGNAL
3410    IF T(M,3)+O(M,3)< OCURMIN-OCURMAX THEN
        X(1,M,3)=TT(1,M)+OCURMAX:Y(1,M,3)=TT(1,M)+OCURMIN-T
        (M,3)-O(M,3) ELSE
        X(1,M,3)=TT(1,M)+OCURMIN-O(M,3)-T(M,3):Y(1,M,3)=TT(
        1,M)+OCURMAX
3420    Z(1,M,3)=O(M,3)+T(M,3)-TT(1,M)-OCURMAX
3430    IF I(M,3)+T(M,3)<ICURMIN-ICURMAX THEN
        X(2,M,3)=C-TT(N,1)+TT(N,M)+ICURMAX-O(M,3):Y(2,M,3)=
        C-TT(N,1)+TT(N,M)+ICURMIN-O(M,3)-I(M,3)-T(M,3) ELSE
        X(2,M,3)=C-TT(N,1)+TT(N,M)+ICURMIN-O(M,3)-I(M,3)-T(
        M,3):Y(2,M,3)=C-TT(N,1)+TT(N,M)+ICURMAX-O(M,3)
3440    Z(2,M,3)=O(M,3)+I(M,3)+T(M,3)-TT(N,M)+TT(N,1)-C-ICU
        RMAX
3450    REM LAG-LEAD ON MIDDLE SIGNAL
3460    IF T(M,4)+O(M,4)< OCURMIN-OCURMAX THEN
        X(1,M,4)=TT(1,M)+OCURMAX-I(M,4):Y(1,M,4)=TT(1,M)+OC
        URMIN-T(M,4)-O(M,4)-I(M,4) ELSE
        X(1,M,4)=TT(1,M)+OCURMIN-O(M,4)-T(M,4)-I(M,4):Y(1,M
        ,4)=TT(1,M)+OCURMAX-I(M,4)
3470    Z(1,M,4)=O(M,4)+T(M,4)+I(M,4)-TT(1,M)-OCURMAX
```

```
3480    IF I(M,4)+T(M,4)<ICURMIN-ICURMAX THEN
        X(2,M,4)=C-TT(N,1)+TT(N,M)+ICURMAX:Y(2,M,4)=C-TT(N,
        1)+TT(N,M)+ICURMIN-I(M,4)-T(M,4) ELSE
        X(2,M,4)=C-TT(N,1)+TT(N,M)+ICURMIN-I(M,4)-T(M,4):Y(
        2,M,4)=C-TT(N,1)+TT(N,M)+ICURMAX
3490    Z(2,M,4)=I(M,4)+T(M,4)-TT(N,M)+TT(N,1)-C-ICURMAX
3500    FOR J=1 TO 4
3501        IF X(1,M,J)>C THEN X(1,M,J)=X(1,M,J) MOD
            C:Y(1,M,J)=Y(1,M,J) MOD C:Z(1M,J)=Z(1,M,J)+C
3502        IF X(2,M,J)>C THEN X(2,M,J)=X(2,M,J) MOD
            C:Y(2,M,J)=Y(2,M,J) MOD C:Z(2,M,J)=Z(2,M,J)+C
3503        IF Y(1,M,J)>C THEN Y(1,M,J)=C
3504        IF Y(2,M,J)>C THEN Y(2,M,J)=C
3510    NEXT J
3520    RETURN
3530    REM  RESET BEG AND END OF BANDS MOD CYCLE LENGTH
3550    IF EOB(M)-TT(1,M) <0 THEN
        EOB(M)=EOB(M)+C:BOB(M)=BOB(M)+C:GOTO 3550
3570    IF EIB(M)-TT(N,M)+TT(N,1)-C <0 THEN
        EIB(M)=EIB(M)+C:BIB(M)=BIB(M)+C:GOTO 3570
3580    IF BOB(M)-TT(1,M) > OCURMAX THEN
        OCURMAX=BOB(M)-TT(1,M)
3590    IF EOB(M)-TT(1,M) < OCURMIN THEN
        OCURMIN=EOB(M)-TT(1,M)
3600    IF BIB(M)-TT(N,M)+TT(N,1)-C > ICURMAX THEN
        ICURMAX=BIB(M)-TT(N,M)+TT(N,1)-C
3610    IF EIB(M)-TT(N,M)+TT(N,1)-C < ICURMIN THEN
        ICURMIN=EIB(M)-TT(N,M)+TT(N,1)-C
3620    RETURN
3630    REM DETERMINE MAX ON MID SIGNALS
3640    OPTBAND=-1
3650    FOR E = 1 TO 4
3660        IF OPTBAND> GRBD(M,E) THEN GOTO 3700
3670        OPTBAND=GRBD(M,E)
3680        OPTLB = VARB(M,E)
3690        OPTUP = VARE(M,E)
3700    NEXT E
3710    FOR E = 1 TO 4
3720        IF OPTBAND=GRBD(M,E) THEN GOSUB 3750
3730    NEXT E
3740    RETURN
3750    LPRINT "OFFSET RANGE FROM
        ";VARB(M,E);"TO";VARE(M,E);"YIELDS MAX SUM OF
        BANDS";GRBD(M,E)
3760    IF E =1 THEN LPRINT "LAG-LAG AT
        SIGNAL";M:OFFSET(M)=VARB(M,E):OPTPAT(M)=1
3770    IF E =2 THEN LPRINT "LEAD-LEAD AT
        SIGNAL";M:OFFSET(M)=VARB(M,E):OPTPAT(M)=2
3780    IF E =3 THEN LPRINT "LEAD-LAG AT
        SIGNAL";M:OFFSET(M)=VARB(M,E):OPTPAT(M)=3
3790    IF E =4 THEN LPRINT "LAG-LEAD AT
        SIGNAL";M:OFFSET(M)=VARB(M,E):OPTPAT(M)=4
```

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3800    J=M:GOSUB 3240
3810    REM GOSUB 3530
3820    RETURN
3830    REM CALCULATE MAXIMUM GREENBAND AND OFFSETS
3840    FA(H)=0: PS(H)=0:REM CHECK FOR 0 BANDWIDTH
3850    IF A(1,H)<0 THEN A(1,H)=0
3860    IF A(2,H)<0 THEN A(2,H)=0
3870    IF A(1,H)+C(1,H)<0 THEN A(1,H)=0:C(1,H)=0:B(1,H)=C
3880    IF B(2,H)<0 THEN B(2,H)=0
3890    IF A(2,H)+C(2,H)<0 THEN A(2,H)=0:C(2,H)=0:B(2,H)=C
3900    IF B(1,H)<0 THEN B(1,H)=0
3910    IF C(1,H)<0 THEN V(1,H)=-C(1,H) ELSE V(1,H)=0
3920    IF A(1,H)+B(1,H)+C(1,H) < C THEN
        U(1,H)=A(1,H)+B(1,H)+C(1,H) ELSE U(1,H)=C
3930    IF C(2,H)<0 THEN V(2,H)=-C(2,H) ELSE V(2,H)=0
3940    IF A(2,H)+B(2,H)+C(2,H) < C THEN
        U(2,H)=A(2,H)+B(2,H)+C(2,H) ELSE U(2,H)=C
3950    IF V(2,H)<= V(1,H) THEN PS(H)=PS(H)+1000 ELSE IF
        V(2,H)<A(1,H) THEN PS(H)=PS(H)+2000 ELSE IF
        V(2,H)<B(1,H) THEN PS(H)=PS(H)+3000 ELSE IF
        V(2,H)<U(1,H) THEN PS(H)=PS(H)+4000 ELSE
        PS(H)=PS(H)+5000
3960    IF A(2,H)< V(1,H) THEN PS(H)=PS(H)+100 ELSE IF
        A(2,H)<A(1,H) THEN PS(H)=PS(H)+200 ELSE IF
        A(2,H)<B(1,H) THEN PS(H)=PS(H)+300 ELSE IF
        A(2,H)<U(1,H) THEN PS(H)=PS(H)+400 ELSE
        PS(H)=PS(H)+500
3970    IF B(2,H)< V(1,H) THEN PS(H)=PS(H)+10 ELSE IF
        B(2,H)<A(1,H) THEN PS(H)=PS(H)+20 ELSE IF
        B(2,H)<B(1,H) THEN PS(H)=PS(H)+30 ELSE IF
        B(2,H)<U(1,H) THEN PS(H)=PS(H)+40 ELSE
        PS(H)=PS(H)+50
3980    IF U(2,H)< V(1,H) THEN PS(H)=PS(H)+1 ELSE IF
        U(2,H)<A(1,H) THEN PS(H)=PS(H)+2 ELSE IF
        U(2,H)<B(1,H) THEN PS(H)=PS(H)+3 ELSE IF
        U(2,H)<U(1,H) THEN PS(H)=PS(H)+4 ELSE PS(H)=PS(H)+5
3990    IF PS(H)=1111 THEN IF A(1,H)+C(1,H)>A(2,H)+C(2,H)
        THEN MAXB(H)=
        A(1,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=B(1,H) ELSE
        MAXB(H)=A(2,H)+C(2,H):OFFB(H)=A(2,H):OFFE(H)=B(2,H)
4000    IF PS(H)=1112 THEN MAXB(H)=
        A(1,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=B(1,H)
4010    IF PS(H)=1113 THEN MAXB(H)=
        A(2,H)+C(2,H):OFFB(H)=A(2,H):OFFE(H)=B(2,H)
4020    IF PS(H)=1114 THEN MAXB(H)=
        A(2,H)+C(2,H):OFFB(H)=A(2,H):OFFE(H)=B(2,H)
4030    IF PS(H)=1115 THEN MAXB(H)=
        A(2,H)+C(2,H):OFFB(H)=A(2,H):OFFE(H)=B(2,H)
4040    IF PS(H)=1122 THEN MAXB(H)=
        A(1,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=B(1,H)
```

```
4050     IF PS(H)=1123 THEN MAXB(H)=
         A(2,H)+B(2,H)+C(2,H)+C(1,H):OFFB(H)=B(2,H):OFFE(H)=
         A(1,H)
4060     IF PS(H)=1124 THEN MAXB(H)=
         A(2,H)+B(2,H)+C(2,H)+C(1,H):OFFB(H)=B(2,H):OFFE(H)=
         A(1,H)
4070     IF PS(H)=1125 THEN MAXB(H)=
         A(2,H)+B(2,H)+C(2,H)+C(1,H):OFFB(H)=B(2,H):OFFE(H)=
         A(1,H)
4080     IF PS(H)=1133 THEN MAXB(H)=
         A(2,H)+A(1,H)+C(2,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=
         B(2,H)
4090     IF PS(H)=1134 THEN MAXB(H)=
         A(2,H)+A(1,H)+C(2,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=
         B(2,H)
4100     IF PS(H)=1135 THEN MAXB(H)=
         A(2,H)+A(1,H)+C(2,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=
         B(1,H)
4110     IF PS(H)=1144 THEN MAXB(H)=
         A(2,H)+A(1,H)+C(2,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=
         B(1,H)
4120     IF PS(H)=1145 THEN MAXB(H)=
         A(2,H)+A(1,H)+C(2,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=
         B(1,H)
4130     IF PS(H)=1155 THEN MAXB(H)=
         A(2,H)+A(1,H)+C(2,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=
         B(1,H)
4140     IF PS(H)=1222 THEN
         MAXB(H)=A(1,H)+C(1,H):OFFB(H)=A(1,H):OFFE(H)=B(1,H)
4150     IF PS(H)=1223 THEN
         MAXB(H)=A(2,H)+B(2,H)+C(2,H)+C(1,H):OFFB(H)=B(2,H):
         OFFE(H)=A(1,H)
4160     IF PS(H)=1224 THEN
         MAXB(H)=A(2,H)+B(2,H)+C(2,H)+C(1,H):OFFB(H)=B(2,H):
         OFFE(H)=A(1,H)
4170     IF PS(H)=1225 THEN
         MAXB(H)=A(2,H)+B(2,H)+C(2,H)+C(1,H):OFFB(H)=B(2,H):
         OFFE(H)=A(1,H)
4180     IF PS(H)=1233 THEN
         MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(1,H):
         OFFE(H)=B(2,H)
4190     IF PS(H)=1235 THEN
         MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(1,H):
         OFFE(H)=B(2,H)
4200     IF PS(H)=1244 THEN
         MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(1,H):
         OFFE(H)=B(1,H)
4210     IF PS(H)=1245 THEN
         MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(1,H):
         OFFE(H)=B(1,H)
```

```
4220      IF PS(H)=1255 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(1,H):
          OFFE(H)=B(1,H)
4230      IF PS(H)=1333 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(2,H):
          OFFE(H)=B(2,H)
4240      IF PS(H)=1334 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(2,H):
          OFFE(H)=B(2,H)
4250      IF PS(H)=1335 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(2,H):
          OFFE(H)=B(2,H)
4260      IF PS(H)=1344 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(2,H):
          OFFE(H)=B(1,H)
4270      IF PS(H)=1345 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(2,H):
          OFFE(H)=B(1,H)
4280      IF PS(H)=1355 THEN
          MAXB(H)=A(2,H)+C(2,H)+A(1,H)+C(1,H):OFFB(H)=A(2,H):
          OFFE(H)=B(1,H)
4290      IF PS(H)=1444 THEN
          MAXB(H)=A(1,H)+B(1,H)+C(1,H)+C(2,H):OFFB(H)=B(1,H):
          OFFE(H)=A(2,H)
4300      IF PS(H)=1445 THEN
          MAXB(H)=A(1,H)+B(1,H)+C(1,H)+C(2,H):OFFB(H)=B(1,H):
          OFFE(H)=A(2,H)
4310      IF PS(H)=1455 THEN
          MAXB(H)=A(1,H)+B(1,H)+C(1,H)+C(2,H):OFFB(H)=B(1,H):
          OFFE(H)=A(2,H)
4320      IF PS(H)=1555 THEN
          MAXB(H)=A(2,H)+C(2,H):OFFB(H)=A(2,H):OFFE(H)=B(2,H)
4330      IF PS(H)>1555 THEN FA(H)=9:GOSUB 3080 ELSE GOTO
          4350
4340      IF FA(H)=9 THEN GOTO 3830
4350      RETURN
4360      HW1H=W(1,R,T)
4370      HX1H=X(1,R,T)
4380      HY1H=Y(1,R,T)
4390      HQ1H=Q(1,R,T)
4400      HZ1H=Z(1,R,T)
4410      W(1,R,T)=W(2,R,T)
4420      X(1,R,T)=X(2,R,T)
4430      Y(1,R,T)=Y(2,R,T)
4440      Q(1,R,T)=Q(2,R,T)
4450      Z(1,R,T)=Z(2,R,T)
4460      W(2,R,T)=HW1H
4470      X(2,R,T)=HX1H
4480      Y(2,R,T)=HY1H
4490      Q(2,R,T)=HQ1H
4500      Z(2,R,T)=HZ1H
4510      RETURN
```

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BIOGRAPHICAL SKETCH

Veretta Johnson was born in Birmingham, Alabama, on June 5, 1952. She graduated from George Washington Carver High School in Birmingham in May, 1970. In September, 1970, she entered the University of Alabama in Tuscaloosa, Alabama, and received a Bachelor of Science degree in mathematics in December, 1973. She continued at the University of Alabama and received the Master of Arts degree in mathematics in December, 1974.

From July, 1975, through August, 1984, she was employed as a computer programmer and as an assistant professor of mathematics.

She entered the University of Florida in August, 1984 and began work on the Doctor of Philosophy degree with an emphasis on traffic and transportation engineering. She was awarded the McKnight Doctoral Fellowship, a Graduate Research Fellowship with the Federal Highway Administration and worked as a graduate research assistant on various research projects while a graduate student at the University of Florida.

From May, 1988, through August, 1992, she worked as an assistant professor of mathematics while completing research for and writing this dissertation.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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